# THE STRENGTH OF HYPERCUBE 

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Let $G$ be a graph of order $p$. A numbering $f$ of $G$ is a labeling that assigns distinct elements of the set $\{1,2, \ldots, p\}$ to the vertices of $G$, where each edge $u v$ of $G$ is labeled $f(u)+f(v)$. The strength $\operatorname{str}_{f}(G)$ of a numbering $f: V(G) \rightarrow$ $\{1,2, \ldots, p\}$ of $G$ is defined by

$$
\operatorname{str}_{f}(G)=\max \{f(u)+f(v) \mid u v \in E(G)\}
$$

that is, $\operatorname{str}_{f}(G)$ is the maximum edge label of $G$, and the strength $\operatorname{str}(G)$ of a graph $G$ itself is

$$
\operatorname{str}(G)=\min \left\{\operatorname{str}_{f}(G) \mid f \text { is a numbering of } G\right\} .
$$

There are infinitely many graphs $G$ for which $\delta(G) \geq 1$ and $\operatorname{str}(G)=$ $|V(G)|+\delta(G)$.

In this paper, we provide an improved lower bound for $\operatorname{str}\left(Q_{n}\right)$, where $n \geq 4$.

# Domination Subdivision Number of a Graph 

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#### Abstract

Let $G=(V, E)$ be graph. A subset $S$ of $V$ is called a dominating set of $G$ if every vertex in $V-S$ is adjacent to a vertex in $S$. The minimum cardinality of a dominating set of $G$ is called the domination number of $G$ and is denoted by $\gamma(G)$. The domination subdivision number $s d_{\gamma}(G)$ is the minimum number of edges that must be subdivided (each edge can be subdivided at most once) in order to increase the domination number. In this talk we present the current status of the conjectures on this parameter. We also discuss the analogous parameter for secure domination number.


# Decomposition and $r$-hued Coloring of $K_{4}(h)$-minor Free Graphs Murong Xu ${ }^{1}$ <br> Joint work with Ye Chen ${ }^{2}$, Hong-Jian Lai ${ }^{3}$ and Huimin Song ${ }^{4}$ <br> ${ }^{1}$ ) Department of Mathematics, The Ohio State University, Columbus, OH 43210 xu.3646@osu.edu <br> ${ }^{2}$ ) Department of Mathematics and Statistics, Northern Arizona University, Flagstaff, AZ 86011 <br> ${ }^{3}$ ) Department of Mathematics, West Virginia University, Morgantown, WV 26506 <br> ${ }^{4}$ ) School of Mathematics and Statistics, Shandong University, Weihai, Shandong 264209, P.R.China 

A $(k, r)$-coloring of a graph $G$ is a proper $k$-vertex coloring of $G$ such that the neighbors of each vertex of degree $d$ will receive at least $\min \{d, r\}$ different colors. The $r$-hued chromatic number, denoted by $\chi_{r}(G)$, is the smallest integer $k$ for which a graph $G$ has a $(k, r)$-coloring. Let $f(r)=r+3$ if $1 \leq r \leq 2, f(r)=r+5$ if $3 \leq r \leq 7$ and $f(r)=\lfloor 3 r / 2\rfloor+1$ if $r \geq 8$. In [Discrete Math., 315-316 (2014) 47-52], it is conjectured that if $G$ is planar, then $\chi_{r}(G) \leq f(r)$; and verified this conjecture for $K_{4}$-minor free graphs. To further this result, we define $K_{4}(h)$ as a collection of simple graphs obtained from $K_{4}$ by subdividing $h-4$ vertices to $K_{4}$, for an integer $h \geq 4$. In this talk, for $5 \leq h \leq 7$, a decomposition of $K_{4}(h)$-minor free graphs will be demonstrated and its application will be discussed.

Nearly Platonic graphs<br>Dalibor Froncek ${ }^{1}$<br>Joint work with Jiangyi Qiu ${ }^{1}$<br>${ }^{1}$ ) University of Minnesota Duluth dalibor@d.umn.edu

A Platonic graph is a vertex-regular planar graph with all faces of the same size. It is well known that there exist exactly five such graphs: tetrahedron, octahedron, hexahedron, icosahedron, and dodecahedron.

Recently, William Keith asked whether there exist 1-nearly Platonic graphs that would differ from Platonic in just one face. That is, vertex-regular planar graphs with all faces except one having the same size. W. Keith, D. Kreher and the speaker showed that there are no 2-connected 1-nearly Platonic graphs. We extend the non-existence result to graphs with connectivity one.

On the other hand, there are well known classes of 2-nearly Platonic graphs with exactly two exceptional faces, both of the same size. We will and ask (and partially answer) some questions about 2- and 3-nearly Platonic graphs. In other words, about vertex-regular planar graphs with exactly two or three exceptional faces.

Linear bounds on nowhere-zero group irregularity strength and nowhere-zero group sum chromatic number of graphs Sylwia Cichacz ${ }^{1}$
Joint work with Marcin Anholcer ${ }^{2}$, Jakub Przybyło ${ }^{1}$
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We investigate the group irregularity strength, $s_{g}(G)$, of a graph, i.e. the least integer $k$ such that taking any Abelian group of order $k$, there exists a function $f: E(G) \rightarrow$ so that the sums of edge labels incident with every vertex are distinct. So far the best upper bound on $s_{g}(G)$ for a general graph $G$ was exponential in $n-c$, where $n$ is the order of $G$ and $c$ denotes the number of its components. In this talk we show that $s_{g}(G)$ is linear in $n$, namely not greater than $2 n$.

We consider also locally irregular labeling where we require only sums of adjacent vertices to be distinct. For the corresponding graph invariant we prove the general upper bound: $\Delta(G)+\operatorname{col}(G)-1$ (where $\operatorname{col}(G)$ is the coloring number of $G$ ) in the case when we do not use the identity element as an edge label.

# Odd Twists on Strongly Chordal Graphs <br> Terry McKee 

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A graph is chordal iff every cycle long enough to have a chord does have a chord, and a graph is strongly chordal iff it is chordal and every even cycle long enough to have an odd chord-meaning a chord whose endpoints are an odd distance apart in the cycle - does have an odd chord. (Despite this somewhat awkward characterization, strongly chordal can be defended in several ways as the natural strengthening of chordal.)

Now define a graph to be oddly chordal iff it is chordal and every odd cycle long enough to have an odd chord does have an odd chord. Strongly chordal graphs turn out to be oddly chordal, with the oddly chordal graphs characterized by a twist on the classic forbidden subgraph characterization of strongly chordal graphs. Moreover, both strongly chordal and oddly chordal graphs have new, related characterizations in terms of uncrossed chords of appropriate-length cycles.

Hamiltonian Properties in $k$-Partite Graphs Linda Lesniak ${ }^{1}$ Daniela Ferrero ${ }^{2}$

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A best possible sufficient condition for Hamiltonicity, in term of the number of edges, in a $k$-partite graph will be presented. The implications of this result, based on a very recent result of Chen, Gould, Gu and Saito, will also be discussed.

# Chorded cycles in dense graphs 

Xiaofeng Gu ${ }^{1}$<br>Joint work with Guantao Chen ${ }^{2}$, Ronald J. Gould ${ }^{3}$ and Akira Saito ${ }^{4}$<br>${ }^{1}$ Department of Mathematics, University of West Georgia xgu@westga.edu<br>${ }^{2}$ Department of Mathematics and Statistics, Georgia State University<br>${ }^{3}$ Department of Mathematics and Computer Science, Emory University<br>${ }^{4}$ Department of Information Science, Nihon University, Japan

A cycle of length $k$ is called a $k$-cycle. A non-induced cycle is called a chorded cycle. Let $n$ be an integer with $n \geq 4$. A graph $G$ of order $n$ is chorded pancyclic if $G$ contains a chorded $k$-cycle for every integer $k$ with $4 \leq k \leq n$. Cream, Gould and Hirohata have proved that a graph of order $n$ satisfying $\operatorname{deg}_{G} u+\operatorname{deg}_{G} v \geq n$ for every pair of nonadjacent vertices $u, v$ in $G$ is chorded pancyclic, with some exceptional graphs. They have also conjectured that if $G$ is hamiltonian, we can replace the degree sum condition with the weaker density condition $|E(G)| \geq \frac{1}{4} n^{2}$ and still guarantee the same conclusion. We prove this conjecture by showing that if a graph $G$ of order $n$ with $|E(G)| \geq \frac{1}{4} n^{2}$ contains a $k$-cycle,then $G$ contains a chorded $k$-cycle, with some exceptional graphs. We further relax the density condition for sufficient large $k$.

Distance vertex irregular labeling of graphs Slamin ${ }^{1}$<br>${ }^{1}$ ) Informatics Study Program, University of Jember, Indonesia slamin@unej.ac.id

The distance vertex irregular labeling on a graph $G$ with vertex set $V$ is defined to be an assignment $\lambda: V \rightarrow\{1,2, \cdots, k\}$ such that the set of vertex weights consists of distinct numbers, where the weight of a vertex $v$ in $G$ is defined as the sum of the labels of all the vertices adjacent to $v$ (distance 1 from $v)$. The distance irregularity strength of $G$, denoted by $\operatorname{dis}(G)$, is the minimum value of the largest label $k$ over all such irregular assignments. We present the distance vertex irregular labeling and its other types for some particular classes of graphs.

Finding disjoint theta graphs with a given minimum degree Michael Santana ${ }^{1}$<br>Joint work with Emily Marshall ${ }^{2}$<br>${ }^{1}$ Grand Valley State University<br>santanmi@gvsu.edu<br>${ }^{2}$ Arcadia College

In 1999, Kawarabayahsi showed that if $G$ is a graph on exactly $4 k$ vertices with $\delta(G) \geq\left\lceil\frac{5}{2} k\right\rceil$, then $G$ contains $k$ vertex-disjoint theta graphs (i.e., $G$ contains a $K_{4}^{-}$-factor). In 2014, Chiba et al. showed that if $G$ has a large number of vertices $\left(\Omega\left(k^{3 k}\right)\right)$ and $\delta(G) \geq 2 k$, then $G$ also contains $k$ vertex-disjoint theta graphs. Interestingly, the minimum degree conditions in both results are best possible for the number of vertices considered. This leads to a very natural question as to when and how the minimum degree threshold changes from $\left\lceil\frac{5}{2} k\right\rceil$ to $2 k$. In this talk, we extend the result of Kawarabayashi by showing that every graph $G$ on at least $4 k$ vertices with $\delta(G) \geq\left\lceil\frac{5}{2} k\right\rceil$ contains $k$ vertex-disjoint theta graphs. This result turns out to be best possible for $n$-vertex graphs where $4 k \leq n<5 k$, and using a tiling result of Shokoufandeh and Zhao, we discuss when the minimum degree threshold potentially transitions from $\left\lceil\frac{5}{2} k\right\rceil$ to $2 k$.

Inclusive distance vertex irregular labelings Andrea Feňovčíková ${ }^{1}$<br>Joint work with Martin Bača ${ }^{1}$, Slamin ${ }^{2}$ and Kiki A. Sugeng ${ }^{3}$<br>${ }^{1}$ ) Department of Applied Mathematics and Informatics, Technical University, Košice, Slovakia andrea.fenovcikova@tuke.sk<br>${ }^{2}$ ) Information System Study Program, University of Jember, Jember, Indonesia<br>${ }^{3}$ ) Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Kampus UI Depok, Depok, Indonesia

For a simple graph $G$, a vertex labeling $f: V(G) \rightarrow\{1,2, \ldots, k\}$ is called a $k$-labeling. The weight of a vertex $v$, denoted by $w t_{f}(v)$ is the sum of all vertex labels of vertices in the closed neighborhood of the vertex $v$. A vertex $k$-labeling is defined to be an inclusive distance vertex irregular $k$-labeling of $G$ if for every two different vertices $u$ and $v$ there is $w t_{f}(u) \neq w t_{f}(v)$. The minimum $k$ for which the graph $G$ has an inclusive distance vertex irregular $k$-labeling is called the inclusive distance vertex irregularity strength of $G$.

In the talk we will establish some bounds of the inclusive distance vertex irregularity strength and determine the exact value of this parameter for several families of graphs.

The Maximum Rectilinear Crossing Number of Some Spiders<br>Lauren Keough ${ }^{1}$<br>Joint work with Joshua Fallon ${ }^{2}$, Kirsten Hogenson ${ }^{3}$, Mario Lomelí ${ }^{4}$, Marcus Schaefer ${ }^{5}$, and Pablo Soberón ${ }^{6}$<br>${ }^{1}$ ) Grand Valley State University keoulaur@gvsu.edu<br>${ }^{2}$ ) Carenegie Mellon University<br>${ }^{3}$ ) Colorado College<br>${ }^{4}$ ) Universidad Autonoma de San Luis Potosí<br>${ }^{5}$ ) DePaul University<br>${ }^{6}$ ) Baruch College, CUNY

Given a graph, one might ask if we can drawn that graph in the plane without crossings. We can flip that question though and ask, what is the maximum possible number of crossings a given graph $G$ can have. For a graph $G$, the maximum rectilinear crossing number is the maximum number of edge crossings that can appear in a drawing of $G$ in the plane when each edge is drawn straight. Given that non-incident edges can not cross we can compute a trivial upper bound called the thrackle bound. Building on the work of Woodall, who showed that no tree that contains a subgraph isomorphic to $K_{1,3}$ with each edge subdivided once achieves the thrackle bound, we compute the maximum rectinilear crossing number of more general subdivided stars.

Structures and Numbers of Mincuts in Two-Terminal Directed<br>Acyclic Graphs<br>Mark Korenblit ${ }^{1}$<br>Joint work Vadim E. Levit ${ }^{2}$<br>${ }^{1}$ ) Holon Institute of Technology, Israel<br>korenblit@hit.ac.il<br>${ }^{2}$ ) Ariel University, Israel<br>levitv@ariel.ac.il

In this talk, we discuss minimal cuts (mincuts) in two-terminal directed acyclic graphs (st-dags). A special st-dag characterized by a nested structure generated by its mincuts is called nested. We prove that every nested graph is series-parallel. We demonstrate that the minimum possible number of mincuts in an $n$-vertex st-dag is $n-1$. Moreover, if an st-dag is non-series-parallel, then the number of its mincuts must be larger than $n-1$. Our main observation is that an st-dag of order $n$ has exactly $n-1$ mincuts if and only if it is nested. It is shown that a nested graph can be obtained by a parallel composition of a nested graph and a single edge or by a series composition of nested graphs. Using the recursive structure of nested graphs, it is possible to present an algorithm for their recognition.

# Independence Number of Maximal Planar Graphs 

Allan Bickle

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We show that for a maximal planar graph $G$ with order $n \geq 4$, the independence number satisfies $\frac{n}{4} \leq \alpha(G) \leq$ $\frac{2}{3} n-\frac{4}{3}$. We show the lower bound is sharp and characterize the extremal graphs for the upper bound.

Matching Number and Modulo Orientations<br>Jian-Bing Liu ${ }^{1}$<br>Joint work with Jiaao $\mathrm{Li}^{2}$ and Hong-Jian Lai ${ }^{3}$<br>${ }^{1,3}$ Department of Mathematics, West Virginia University, Morgantown, WV 26506<br>${ }^{2}$ Department of Mathematics, Nankai University, Tianjin, 300071, China Email: j10068@mix.wvu.edu(Jian-Bing Liu)

It was conjectured by Jaeger that the family of every $4 p$-edge-connected graphs has modulo $(2 p+1)$-orientation. Thomassen showed that the edge connectivity $2(2 p+1)^{2}+2 p+1$ is enough to guarantee modulo ( $2 p+1$ )-orientation, and it was further improved by Lovász, Thomassen, Wu and Zhang. A graph $G$ is $\left\langle\mathcal{S} \mathbb{Z}_{2 p+1}\right\rangle$-reduced if $G$ does not have any nontrivial strongly $\mathbb{Z}_{2 p+1}$-connected subgraph. In this paper, we show that if a family of graphs has bounded matching number, then there are only finitely many ( $2 p+2$ )-edge-connected $\left\langle\mathcal{S} \mathbb{Z}_{2 p+1}\right\rangle$ reduced graphs without modulo $(2 p+1)$ orientation in this family.

On the Local Metric Dimension of Line Graphs<br>Rinovia Simanjuntak ${ }^{1}$<br>Joint work with Fithri Annisatun ${ }^{2}$

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For an ordered set $W=\left\{w_{1}, w_{2}, \ldots, w_{k}\right\}$ of $k$ distinct vertices in a nontrivial connected graph $G$, the representation of a vertex $v$ of $G$ with respect to $W$ is the $k$-vector $r(v \mid W)=\left(d\left(v, w_{1}\right), d\left(v, w_{2}\right), \ldots, d\left(v, w_{k}\right)\right)$, where $d\left(v, w_{i}\right)$ is the distance between $v$ and $w_{i}$ for $1 \leq i \leq k$. A set $W$ locally resolves of $G$ if $r(u \mid W) \neq r(v \mid W)$ for every pair of adjacent vertices $u, v$ of G . A local resolving set of $G$ of with minimum cardinality is a local metric basis of G , and its cardinality is the local metric dimension, $\operatorname{lmd}(G)$, of $G$.

In this talk, we study the local metric dimension of the line graph $L(G)$ of a nontrivial connected graph $G$. In particular, we establish sharp bounds for $\operatorname{lmd}(L(G))$ in terms of well-known parameters of $G$.

Flows of signed graphs and related topics Zhang Zhang ${ }^{1}$<br>Joint work with Jiaao $\mathrm{Li}^{2}$, You Lu ${ }^{3}$ and Rong Luo ${ }^{4}$, Cun-quan Zhang ${ }^{4}$<br>${ }^{1,4}$ Department of Mathematics, West Virginia University, Morgantown, WV 26506<br>${ }^{2}$ Department of Mathematics, Nankai University, Tianjin, 300071, China<br>${ }^{3}$ Department of Mathematics, Northwestern Polytechnical University, Xian, 710129, China<br>Email: zazhang@mix.wvu.edu(Zhang Zhang)

In 1983, Bouchet proposed a conjecture that every flow-admissible signed graph has a nowhere-zero 6 -flow.This conjecture remains open. Bouchet himself proved that such signed graphs admit nowhere-zero 216-flows; Zýka further proved that such signed graphs admit nowhere-zero 30-flows. Recently, DeVos improved Zýka's result to 12 -flows. In this talk, we further improve DeVos's result and show that every flow-admissible signed graph admits a nowhere-zero 11-flow.

On the Connectedness of 3-Line Graphs<br>Garry L. Johns $1^{1}$<br>Joint work with Khawlah H. Alhulwah and Ping Zhang $2^{2}$<br>${ }^{1}$ ) Saginaw Valley State University<br>glj@svsu.edu<br>${ }^{2}$ ) Western Michigan University

The line graph $L(G)$ is that graph whose vertices are the edges of $G$ where two vertices in $L(G)$ are adjacent if the corresponding edges are adjacent in $G$. For an integer $k \geq 2$, the $k$-line graph of a graph $G$ is the graph whose vertex set is the set of all $k$-paths (paths of order $k$ ) of $G$ where two vertices of the $k$-line graph are adjacent if they are adjacent $k$-paths in $G$. Since the 2 -line graph is the line graph for every graph $G$, this is a generalization of line graphs. In this talk, we focus on 3-line graphs, show that the 3-line graph of $G$ is isomorphic $G$ if and only if $G$ is an odd cycle with at least five vertices, and present several sufficient conditions for the 3 -line graph of a connected graph to be connected.

Cut-edges and regular factors in regular graphs of odd degree Dara Zirlin ${ }^{1}$<br>Joint work with Alexandr V. Kostochka ${ }^{2}$, André Raspaud ${ }^{3}$, Bjarne Toft ${ }^{4}$, and Douglas West ${ }^{5}$.<br>${ }^{1}$ ) University of Illinois at Urbana-Champaign, Urbana IL 61801 zirlin2@illinois.edu<br>${ }^{2}$ ) University of Illinois at Urbana-Champaign, Urbana IL 61801, and Sobolev Institute of Mathematics, Novosibirsk 630090, Russia kostochk@math.uiuc.edu<br>${ }^{3}$ ) Université de Bordeaux, LaBRI UMR 5800, F-33400 Talence, France raspaud@labri.fr<br>${ }^{4}$ ) University of Southern Denmark, Odense, Denmark btoft@imada.sdu.dk<br>${ }^{5}$ ) Zhejiang Normal University, Jinhua, China 321004 and University of Illinois at Urbana-Champaign, Urbana IL 61801<br>dwest@math.uiuc.edu

Previously, Hanson, Loten, and Toft proved that every $(2 r+1)$-regular graph with at most $2 r$ cut-edges has a 2 -factor. We generalize their result by proving for $k \leq(2 r+1) / 3$ that every $(2 r+1)$-regular graph with at most $2 r-3(k-1)$ cut-edges has a $2 k$-factor. We show that the restriction on $k$ and the restriction on the number of cut-edges are sharp.

# On $s$-hamiltonian $Z_{8}$-free line graphs 


#### Abstract

A graph $G$ is $s$-hamiltonian if the removal of at most $s$ vertices from $G$ results in a hamiltonian graph. Let $Z_{8}$ denote the graph derived from identifying one end vertex of a path with 9 vertices with one vertex of a triangle. In [J. of Graph Theory, 664 (2010), 1-11] it is shown that every 3 -connected $Z_{8}$-free line graph is hamiltonian. We proved for any integer $s \geq 1$, a $Z_{8}$-free line graph $L(G)$ is $s$-hamiltonian if and only if $\kappa(L(G)) \geq s+2$.


# Some sharp results on the generalized Turán numbers Yu Qiu ${ }^{1}$ <br> Joint work wit Jie $\mathrm{Ma}^{2}$ <br> ${ }^{1}$ ) Yu Qiu, University of Science and Technology of China yuqiu@mail.ustc.edu.cn <br> ${ }^{2}$ ) Jie Ma, University of Science and Technology of China 

For graphs $T, H$, let ex $(n, T, H)$ denote the maximum number of copies of $T$ in an $n$-vertex $H$-free graph. We prove some sharp results on this generalization of Turán numbers, where our focus is for the graphs $T, H$ satisfying $\chi(T)<$ $\chi(H)$. Erdős generalized the celebrated Turán's theorem by showing that for any $r \geq m$, the Turán graph $T_{r}(n)$ uniquely attains $\operatorname{ex}\left(n, K_{m}, K_{r+1}\right)$. For general graphs $H$ with $\chi(H)=r+1>m$, Alon and Shikhelman showed that $\operatorname{ex}\left(n, K_{m}, H\right)=\binom{r}{m}\left(\frac{n}{r}\right)^{m}+o\left(n^{m}\right)$. Here we determine this error term $o\left(n^{m}\right)$ up to a constant factor. We prove that $\operatorname{ex}\left(n, K_{m}, H\right)=\binom{r}{m}\left(\frac{n}{r}\right)^{m}+\operatorname{biex}(n, H)$. $\Theta\left(n^{m-2}\right)$, where $\operatorname{biex}(n, H)$ is the Turán number of the decomposition family of $H$. As a special case, we extend Erdős' result, by showing that $T_{r}(n)$ uniquely attains ex $\left(n, K_{m}, H\right)$ for any edge-critical graph $H$. We also consider $T$ being non-clique, where even the simplest case seems to be intricate. Following from a more general result, we show that for all $s \leq t, T_{2}(n)$ maximizes the number of $K_{s, t}$ in $n$-vertex triangle-free graphs if and only if $t<s+\frac{1}{2}+\sqrt{2 s+\frac{1}{4}}$.

# Laplacian spectra of a graph and Brouwer's conjecture 

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#### Abstract

For a simple graph $G(V, E)$ with order $n$ and size $m$ having vertices set $v_{1}, v_{2}, \ldots, v_{n}$, the adjacency matrix $A=\left(a_{i j}\right)$ of $G$ is a ( 0,1 )-square matrix of order $n$ whose $(i, j)$-entry is equal to 1 if $v_{i}$ is adjacent to $v_{j}$ and equal to 0 , otherwise. If $D(G)=\operatorname{diag}\left(d_{1}, d_{2}, \ldots, d_{n}\right)$ is the diagonal matrix associated to $G$, the matrix $L(G)=D(G)-A(G)$ is the Laplacian matrix and its spectrum is the Laplacian spectrum of $G$. With $0=\mu_{n} \leq \mu_{n-1} \leq \cdots \leq \mu_{1}$ as the Laplacian spectrum of $G$, let $S_{k}(G)=\sum_{i=1}^{k} \mu_{i}, k=1,2, \ldots, n$ be the sum of $k$ largest Laplacian eigenvalues of $G$. For a graph $G$, Andries Brouwer conjectured that $S_{k}(G)=$ $\sum_{i=1}^{k} \mu_{i} \leq m+\binom{k+1}{2}$ for any $k, k=1,2, \ldots, n$. The Laplacian energy of a graph $G$ as put forward by Gutman and Zhou (2006) is defined as $L E(G)=\sum_{i=1}^{n}\left|\mu_{i}-\frac{2 m}{n}\right|$. However, Brouwer's conjecture remains open at large and if the conjecture is true then it is expected that progress can be made in many fundamental problems in spectral graph theory. In particular, the problem of finding the graph with largest Laplacian energy can be solved. It is noteworthy that any progress on the upper bounds of $S_{k}(G)$ implies a possibility of the verification of Brouwer's conjecture for certain families of graphs and as a consequence gives significant information about Laplacian energy of those families of graphs.

In this talk, we discuss the recent developments on the bounds for $S_{k}(G)$, Brouwer's conjecture and the Laplacian energy of graphs.


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The Super Slater Number of a Graph<br>Jeremy Lyle ${ }^{1}$<br>${ }^{1}$ Olivet Nazarene University<br>sjlyle@olivet.edu

In this talk, we introduce the super slater number of a graph in order to provide lower bounds on the domination of regular graphs. In particular, we provide a family of $\delta$-regular graphs on $n$ vertices for which the super slater number of the graph $G$ provides a lower bound on the domination number of $G$. that can be made arbitrarily larger than the bound $\lceil n /(\delta+1)\rceil$.

# MIGHTY Abstract 

Grace McCourt

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## A bounded diameter strengthening of Ryser's conjecture

Ryser conjectured for any graph $G$ and any integer $r \geq 2$, that in any $r$-coloring of the edges of $G$, there exist at most $(r-1) \alpha(G)$ monochromatic components which cover the vertices of $G$. A stronger version of this conjecture asks whether the vertices of each such graph can be covered by at most $(r-1) \alpha(G)$ monochromatic subgraphs of bounded diameter. This talk will present results on $K_{n}, K_{m, n}$, and graphs $G$ with $\alpha(G)=2$.

# Lozenge tilings of a symmetric hexagon with a missing shamrock Ranjan Rohatgi ${ }^{1}$ <br> Tri Lai ${ }^{2}$ <br> ${ }^{1}$ Saint Mary's College, Notre Dame, IN <br> rrohatgi@saintmarys.edu <br> ${ }^{2}$ University of Nebraska - Lincoln, Lincoln, NE 

MacMahon showed that the number of plane partitions contained in a box is given by a simple product formula. This formula also enumerates the lozenge tilings of hexagons with opposite sides of equal integer length and interior angles of 120 degrees, or equivalently, the number of perfect matchings of the dual graph of such a region. In this talk, we show that in the case of symmetric hexagons, we always have a simple product formula for the number of tilings when removing a shamrock at any position along the symmetry axis, complementing results of Ciucu and Krattenthaler on arbitrary hexagons, and extending work of Eisenkölbl.

# Additive List Coloring of Planar Graphs with Given Girth Jennifer White ${ }^{1}$ <br> Joint work with Axel Brandt ${ }^{2}$ and Sogol Jahanbekam ${ }^{3}$ <br> ${ }^{1}$ ) Saint Vincent College <br> jennifer.white@stvincent.edu <br> ${ }^{2}$ ) Northern Kentucky University ${ }^{3}$ ) Rochester Institute of Technology 

An additive coloring of a graph $G$ is a labeling of the vertices of $G$ from $\{1,2, \ldots, k\}$ such that two adjacent vertices have distinct sums of labels on their neighbors. The least integer $k$ for which a graph $G$ has an additive coloring is called the additive coloring number of $G$, denoted $\chi_{\Sigma}(G)$. In this paper, we improve the current bounds on the additive coloring number for particular classes of graphs by proving results for a list version of additive coloring. We apply the discharging method and the Combinatorial Nullstellensatz to show that every planar graph $G$ with girth at least 5 has $\chi_{\Sigma}(G) \leq 19$, and for girth at least 6,7 , and $26, \chi_{\Sigma}(G)$ is at most 9,8 , and 3 , respectively.

