

# Old and New Generalizations of Line Graphs

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## Abstract

The *line graph*  $L(G)$  of a graph  $G$  is defined to have as its vertices the edges of  $G$ , with two being adjacent if the corresponding edges share a vertex in  $G$ . Line graphs have a rich history. The name line graph was first used by Harary and Norman in 1960. But line graphs were the subject of investigation as far back as 1932 in a paper by H. Whitney in which he showed that for connected graphs, edge-isomorphism implies isomorphism except for  $K_3$  and  $K_{1,3}$ . The line graph transformation is one of the most widely studied of all graph transformations. The concept has been rediscovered several times, with different names such as derived graph, interchange graph, and edge-to-vertex dual. Line graphs can also be considered as intersection graphs.

Several variations and generalizations of line graphs have been proposed and studied. These include the concepts of total graphs, path graphs and others. More recent generalizations include the concept of super line graphs. In a series of several papers, Bagga, Beineke and Varma investigated many properties of super line graphs. Recent work also includes the study of algebraic properties of super line graphs by Bagga and Ferrero. In this presentation we describe these and some other recent generalizations such as triangle graphs.