Create Silent Movies
to Model the Real World
with the TI-84

29th Annual T³ International Conference
Chicago, Illinois

Saturday, March 11, 2017
8:30 a.m. – 10:00 a.m.
Wright, West Tower, Third Floor

John LaMaster, National T³ Instructor
Indiana University Purdue University at Fort Wayne
Department of Mathematical Sciences
2101 Coliseum Blvd. East, Kettler 264
Fort Wayne, IN 46805-1445
E-mail: lamaster@ipfw.edu
260-481-5430

Stuart Moskowitz, National T³ Instructor
Humboldt State University
Mathematics Department
1 Harpst St
Arcata, CA 95521
E-mail: stuart@humboldt.edu
707-445-5795

This handout and related accessories (programs, image files, graphical databases) can be downloaded at http://users.ipfw.edu/lamaster/technology

Table of Contents

Lights! Camera! Action! – Three Ways to Produce Movies on the TI-84 Plus Family Calculator

1. **Live Action**
   Using parametric mode, many classical story problems from traditional algebra and precalculus textbooks can be brought to life by entering the settings in four menus: Mode, Format, Y=, and Window. In the activities *Harry and Sally* and *Batman and the Joker*, the teacher directs the activity from the display calculator live before the class (with some prior setup). Students mathematize the problem to tell the teacher what value of Tmax will stop the movie to show when the main characters meet. The activity *Ditch Diggers* is similar. The student activity *Alexander Hamilton and Aaron Burr* is designed so the settings could be entered by the students to create the movie.

2. **Function Transformations and Graph Styles saved with a Graphical Database** (GDB)
   A graph database (GDB) stores the set of elements that defines a particular graph.
   You can recreate the graph from these elements. You can store up to 10 GDBs in variables GDB0 through GDB9 and recall them to recreate graphs.

   A GDB stores five elements of a graph
   * Graphing mode
   * Window variables
   * Format settings
   * All functions in the Y= editor and the selection status of each
   * Graph style for each Y= function
   GDBs do not contain drawn items (created from the Draw Menu) or stat plot definitions.

   Use 2nd Draw and press the right arrow to the STO menu to enter the command `StoreGDB 1` on the Home Screen to store GDB 1. To recall it, enter the command `RecallGDB 1`.
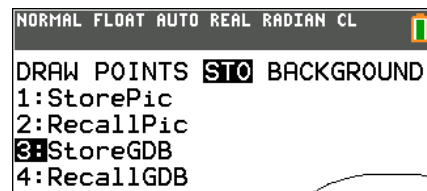
   Some disadvantages with GDB's:
   * Limited to 10 GDB's. However, you can store them in a group file, and there is no limit to the number of group files you may have (up to calculator memory).
   * It is easy to accidentally overwrite a GDB by storing a new one over an existing one.
   * Cannot be given descriptive names so it is difficult to know what is in it.
     TIP: When you store them onto a computer, you can rename the file.
     You can assign a name to each GDB and then create a program to index them.
   * While GDBs store most Mode and Format settings, it does not store Degree vs Radian Mode.
   Some examples using GDB's are *Bus Chase*, *Falling Objects on Different Planets*, *Home Run?* and *Roller Coaster.*

3. **Use a program**
   There is no limit to the number of programs you may have (up to calculator memory) and these can be given somewhat descriptive names. Much more control is possible, in particular, setting mode to Degree or Radians, computer generated imagery (CGI) such as random animated fireworks and flashing lights (*Math 99 Animated Movie*) or replaying automatically using an infinite loop (*Math 99 Animated Movie* and *Spotlight Crawling on a Wall*).
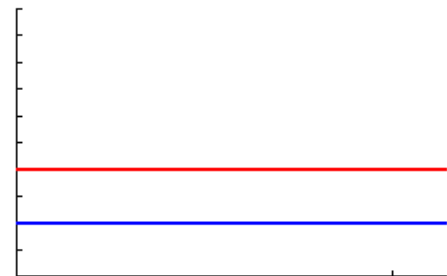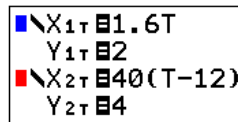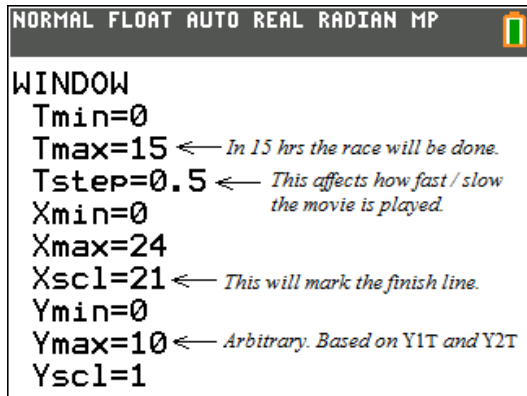
Harry and Sally

Adapted from an Algebra 1 Textbook: Harry the Hare and Sally the Tortoise have a race.
Sally travels at a slow but steady speed of 1.6 mph. Harry travels at constant speed of 40 mph,
but gives Sally a 12 hour head start. If the finish line is 21 miles away, who will win the race?

We will make a silent movie of this scenario to bring this iconic fable to life.
Let Harry travel along the line $y = 4$ and Sally travel along the line $y = 2$.

1. Press MODE.
   Highlight parametric mode and simultaneous graphing.

```
NORMAL FLOAT AUTO REAL RADIAN MP
[ENTER] TO SET CLOCK
MATHPRINT  CLASSIC
NORMAL  SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
FUNCTION  PARAMETRIC  POLAR  SEQ
THICK  DOT-THICK  THIN  DOT-THIN
SEQUENTIAL  SIMUL
REAL  a+bi  re^(θi)
FULL  HORIZONTAL  GRAPH-TABLE
FRACTION TYPE: n/d  Un/d
ANSWERS: AUTO  DEC
STAT DIAGNOSTICS: OFF  ON
STAT WIZARDS: ON  OFF
SET CLOCK  03/11/17 08:30 AM
LANGUAGE:        ENGLISH
```

2. Press WINDOW and enter these settings. Press Y= and enter these equations. Press GRAPH.

```
NORMAL FLOAT AUTO REAL RADIAN MP

WINDOW
 Tmin=0
 Tmax=15   ← In 15 hrs the race will be done.
 Tstep=0.5 ← This affects how fast / slow
 Xmin=0          the movie is played.
 Xmax=24
 Xscl=21   ← This will mark the finish line.
 Ymin=0
 Ymax=10   ← Arbitrary. Based on Y1T and Y2T
 Yscl=1
```

```
\X₁ᴛ=1.6T
 Y₁ᴛ=2
\X₂ᴛ=40(T-12)
 Y₂ᴛ=4
```

Use the 21 mile marker for the finish line.
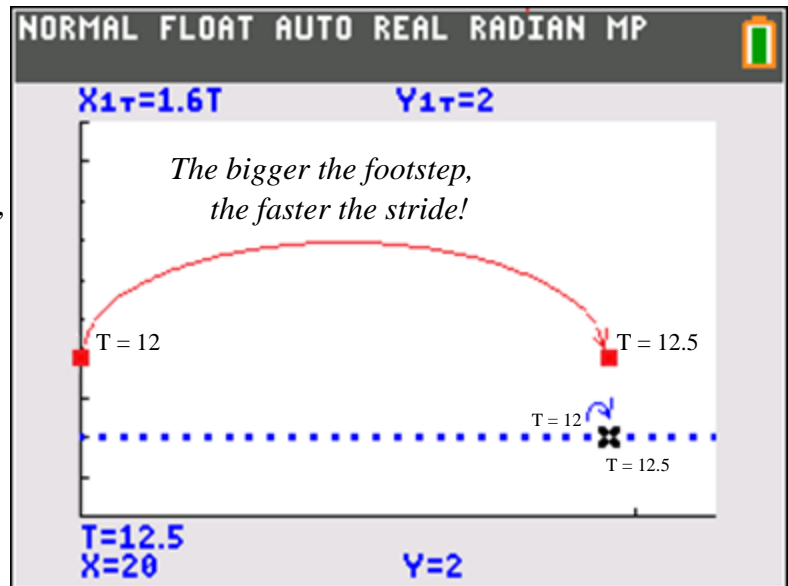
A Look Behind the Scenes: What is happening?

Press MODE, highlight DOT-THICK.

Press GRAPH. Press TRACE and >.
Type 12 to jump to T= 12 hours.

Press ; to move between graphs.
In THICK (or CONNECTED) Graphing,
the TI-84 connects these dots with lines.

Footsteps for each graph are connected
simultaneously, so Harry will graph
remarkably faster than Sally.

Lowering Tstep adds more footprints.
This will slow down the action.

```
NORMAL FLOAT AUTO REAL RADIAN MP
X₁ᴛ=1.6T          Y₁ᴛ=2

        The bigger the footstep,
          the faster the stride!

 T = 12                          T = 12.5

                        T = 12
                        T = 12.5

T=12.5
X=20              Y=2
```

In Dot mode, TRACE from T = 12 to T = 12.5 with TStep = 0.5.
Compare the leaps between footprints for the Hare and the Tortoise.

Set Tstep = 0.1. The distance between footprints is smaller and more footprints appear.
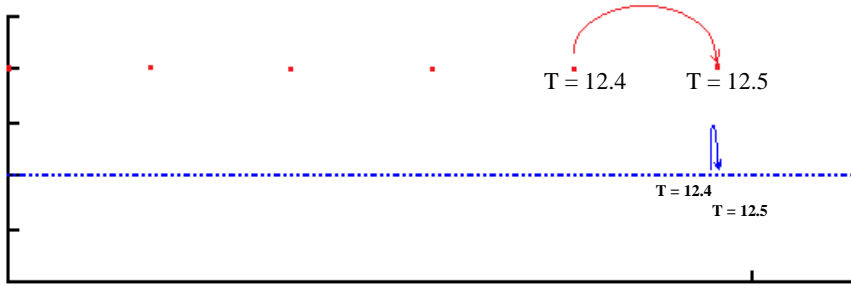
In Dot mode, TRACE from T = 12.4 to T = 12.5 in steps of TStep = 0.1.

## Class Activity: Use the movie to heighten student engagement

Set up your calculator as in Step 2 (with THICK or CONNECTED and Tstep = 0.5).
Present the problem to students. Display to the class the movie on your calculator.
(Students are not using their calculator in this activity to create the movie.).

TIP: To replay the movie without disclosing the formulas in the Y= Editor

1. Press GRAPH
   (if not already on the Graph Screen).
2. Press DRAW.
3. Press 1:ClrDraw.
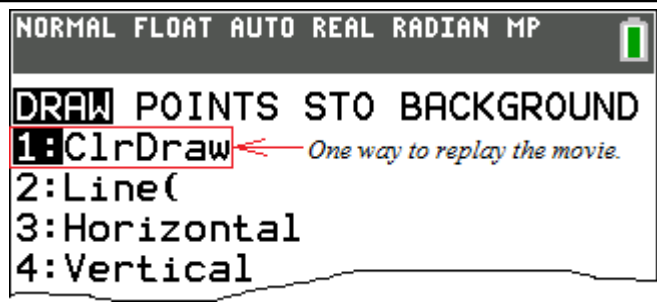
- Who won? Set Tstep = 0.05 to slow down the movie. Press GRAPH.
- Find when they meet, i.e., *When Harry met Sally* ☺
  Replay and ask the class to tell you when to pause the action when they meet (by pressing ENTER).
  (This usually is not easy without Jedi skills.)
  Easier: Have the class write formulas for each racer and solve equations algebraically:
  $$1.6t = 40(t - 12)$$
  $$1.6t = 40t - 480$$
  $$38.4t = 480$$
  $$t = \frac{480}{38.4} = 12.5$$
  Assign Tmax = 12.5, Tstep = 0.5, and press GRAPH.

- *Optional:* ask the class to generate additional questions.
  o At what mile marker did they meet?
    Answer: 20 miles

  o How long did it take each to cross the finish line?
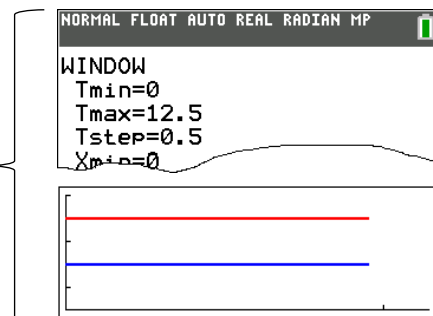   **Answer:** Sally took 13.125 hr =13 hr, 7.5 min;
    Harry took 12.525 hr = 12 hr, 31.5 min

  o What are the values of *t* for which Harry is in the lead? How many seconds is this?
    Answer: From t = 12.5 to t = 12.525.
    This was the last 0.025 hr = 1.5 min = 90 seconds of the race!

Batman and the Joker

This problem is similar to the Harry and Sally problem in that you can direct it from the Teacher Calculator while engaging students in the discussion.

Batman is traveling 200 mph due East when he sees the Joker coming directly toward him 27 miles away, traveling at 70 mph West. How long until they meet?
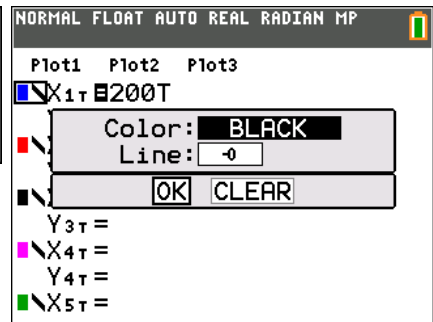
1. Set up your TI-84CE.
    Highlight parametric mode and simultaneous mode.

2. Press Y= and enter these equations:

    Optional: Use the spinner to change graph color (Black/Magenta) and graph style (Path).

    $X_{1T} = 200T$
    $Y_{1T} = 4$
    $X_{2T} = 27 - 70T$
    $Y_{2T} = 4$

    NORMAL FLOAT AUTO REAL RADIAN MP

    Plot1 Plot2 Plot3
    $X_{1T} = 200T$
    Color: **BLACK**
    Line: ◦
    OK CLEAR
    $Y_{3T} =$
    $X_{4T} =$
    $Y_{4T} =$
 $X_{5T} =$

3. Set the window.

    Tmin=0
    Tmax=0.2 *(Set Tmax to be the length of time, in hours, you want to see them driving)*
    Tstep=0.005 *(adjust Tstep to taste; the higher the Tstep, the faster the movie.)*
    Xmin=0
    Xmax=27 *(set Xmax to be the initial distance they are apart.)*
    Xscl=1
    Ymin=0
    Ymax=10
    Yscl=1

    Change the above movie to play for Tmax = 0.05 hours (*or consider using 60ths of an hour*) Show the class that the movie stopped too soon. Bummer.

4. Now ask, "When shall I stop the clock so that the movie ends **exactly** when they meet?"
    Students set up the equation $200t = 27 - 70t$ and solve for *t* algebraically.

5. Once the solution *t* = 0.1 is found, change Tmax = 0.1. Press GRAPH to replay.

    NORMAL FLOAT AUTO REAL RADIAN MP
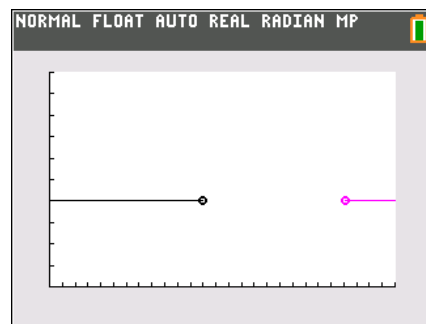
Here's another one, similar to the above:
Batman's speed is 185 mph, Joker's speed is 55 mph and they are initially 60 miles apart.
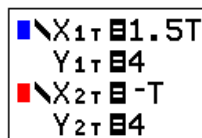
Alexander Hamilton and Aaron Burr

Alex and Aaron meet on the dueling ground, each holding a Super Soaker water pistol. They stand back to back and simultaneously walk until they are 20 ft apart. Alex walks 1 ft per sec West, while Aaron (a bit jittery) walks 1.5 ft per sec East. How many seconds before the distance between them is 20 ft?

First set up your TI-84CE to simulate and explore.

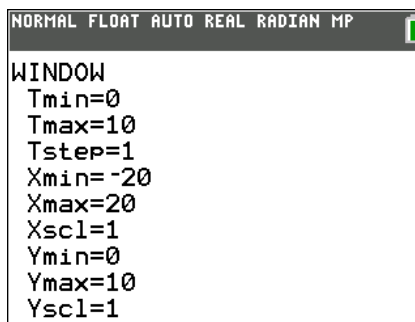1. Highlight parametric mode and simultaneous mode.

    NORMAL FLOAT AUTO REAL RADIAN MP
    [ENTER] TO SET CLOCK
    MATHPRINT CLASSIC
    NORMAL SCI ENG
    FLOAT 0 1 2 3 4 5 6 7 8 9
    RADIAN DEGREE
    FUNCTION PARAMETRIC POLAR SEQ
    THICK DOT-THICK THIN DOT-THIN
    SEQUENTIAL SIMUL
    REAL a+bi re^(θi)
    FULL HORIZONTAL GRAPH-TABLE
    FRACTION TYPE: n/d Un/d
    ANSWERS: AUTO DEC
    STAT DIAGNOSTICS: OFF ON
    STAT WIZARDS: ON OFF
    SET CLOCK 03/11/17 08:30AM
    LANGUAGE: ENGLISH

2. Press Y= to enter equations.

    $X_{1T} = 1.5T$
    $Y_{1T} = 4$
    $X_{2T} = -T$
    $Y_{2T} = 4$

3. After 10 seconds, how far apart are they? _____ ft

    Aaron Burr walked _____ ft.
    Alex Hamilton walked _____ ft.

    Press WINDOW and use these settings.
    You can confirm with TRACE or a TABLE.

    NORMAL FLOAT AUTO REAL RADIAN MP

    WINDOW
     Tmin=0
     Tmax=10
     Tstep=1
     Xmin=-20
     Xmax=20
     Xscl=1
     Ymin=0
     Ymax=10
     Yscl=1

4. In *t* seconds, how far apart are they? _____ (Your answer involves *t*.)

    Aaron Burr walked _____ ft. (Your answer involves *t*.)
    Alex Hamilton walked _____ ft. (Your answer involves *t*.)

5. Write an equation to determine the value of *t* which makes their distance apart equal to 20 ft.
    Solve the equation. Set Tmax equal to this value of *t* and press GRAPH.

    Complete:
    They are 20 ft apart in *t* = ___ seconds;
    Aaron Burr walked _____ ft and Alex Hamilton walked _____ ft

Ditch Diggers

Dan Meyer incorporates video at http://threeacts.mrmeyer to teach with three-act math tasks. (See http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/ for his approach.)
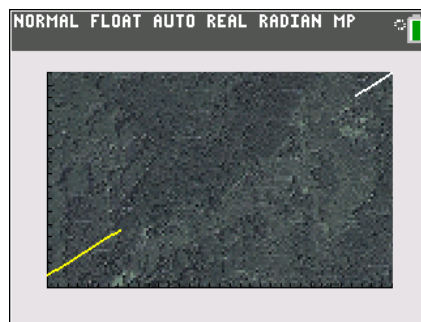
Act One:
Introduce the story/task using as few words as possible.

In this example:
Two ditch diggers start digging towards each other from opposite directions.

The three acts for this movie are available at http://threeacts.mrmeyer.com/ditchdiggers/; however, this movie can be simulated on the TI-84 Family calculator.

See *Dig In to the Real World with the TI-84 Plus CE* at http://users.ipfw.edu/lamaster/technology/ from the 27th Annual T[3] International Conference for details on how to incorporate the calculator into this activity.

Act Two: Coordinates are given for Digger 1 (X_{1T}, Y_{1T}) and Digger 2 (X_{2T}, Y_{2T}) for the first five days to develop the following formulas:
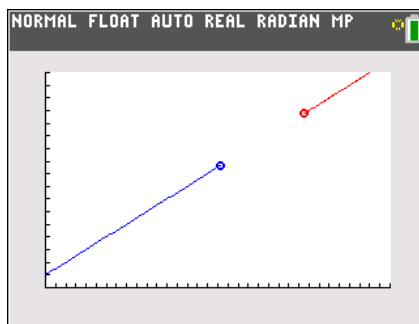
Act Three: Complete the task. Set up a sequel/extension.
Spoiler: The diggers do not meet.

Prequel?

Bus Chase

This problem was published in COMAP Consortium Number 42 in Summer 1992.

A man who is late for work is approaching a bus. When he is 25 meters from the bus it starts to pull away with an acceleration of 1.0 meter per second per second. He sees this and begins to run at 6 meters per second. Will he catch the bus? If not, how close will he come?

First we will represent the position of the man and the bus (relative to the man's starting point) on vertical lines on the screen. After t seconds the man's distance from his starting point is $6t$ and the bus's distance from the man's starting point is $25 + 0.5t^2$.

1. Use X_{1T} and Y_{1T} to represent the man's motion and X_{2T} and Y_{2T} to represent the bus's motion. Make the man and the bus move on the vertical line $x = -3$. Set the equations and the window to do this. Use SIMUL mode. Use O Line Style for the bus and dotted line style for the man.
2. Trace to see if he catches the bus.
3. Graph the distance between the bus and the man versus time t. To do this let $X_{3T} = T$ and $Y_{3T} = Y_{1T} - Y_{2T}$. Keep the window set so that you can see the vertical motions and the plot of Y_{3T}.

Screenshots from the **Bus Chase** problem:
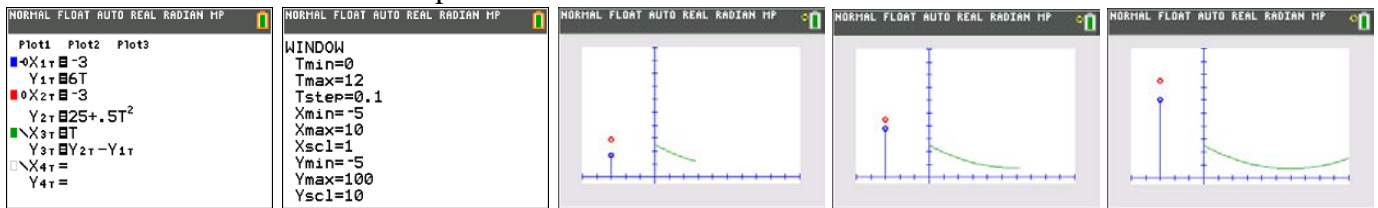
Falling Objects on Different Planets

Teacher: "Three balls will be dropped from a height of 100 meters on each of the planets, Mars, Earth, and Jupiter."
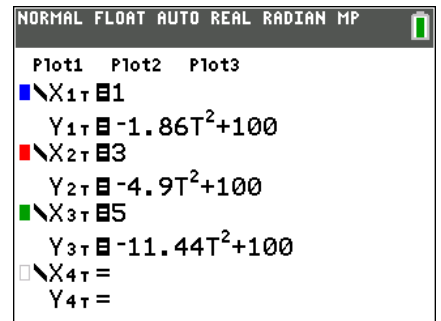*Show the TI-84 movie of the balls dropping. Do not show formulas yet.*
    "Discuss in your groups:
    What is similar about each? What is different?
    Which planet goes with which graph?"
Prior to class set up the calculator using the screens to the right.
A GDB is an excellent way to have this ready to go for class.

Students may bring up the following similarities:
- Each has the same initial height.
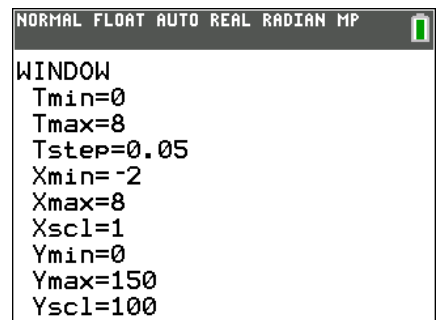- As they fall, they move faster and faster.

Students may bring up the following differences:
- Length of time to drop.
- Rate of acceleration.

Match:
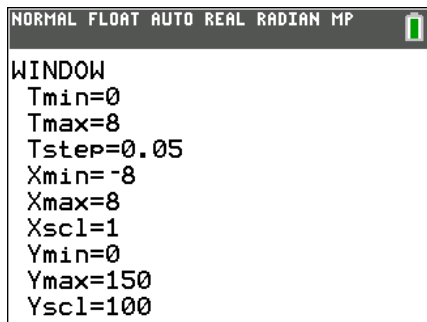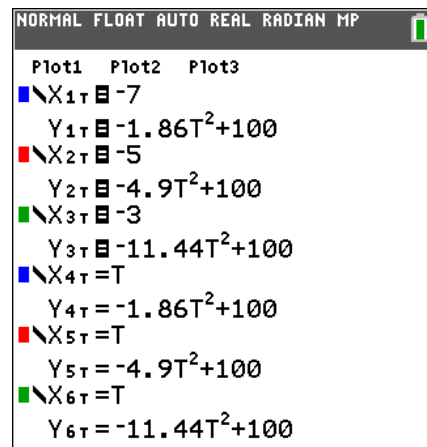The slowest is Jupiter, fastest is Mars, and Earth is in between.
The larger the planet's mass, the larger the acceleration due to gravity.

Optional (based on the level of your students): Help students construct the formulas for each.

Falling Objects on Different Planets (Alternative)

Based on students' level, interest, or available class time, the previous activity can be extended. Instead of setting up the calculator with only the three balls dropping, use the screens to the right, where the last three graphs are deselected (not highlighted) and use the more expanded viewing window $-8 \le x \le 8$.

```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\X₁ᴛ🄱-7
   Y₁ᴛ🄱-1.86T²+100
■\X₂ᴛ🄱-5
   Y₂ᴛ🄱-4.9T²+100
■\X₃ᴛ🄱-3
   Y₃ᴛ🄱-11.44T²+100
■\X₄ᴛ=T
   Y₄ᴛ=-1.86T²+100
■\X₅ᴛ=T
   Y₅ᴛ=-4.9T²+100
■\X₆ᴛ=T
   Y₆ᴛ=-11.44T²+100
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW
 Tmin=0
 Tmax=8
 Tstep=0.05
 Xmin=-8
 Xmax=8
 Xscl=1
 Ymin=0
 Ymax=150
 Yscl=100
```
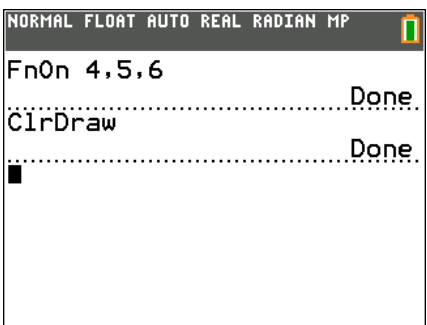
As before, play the movie while asking students to find similarities, differences, and to match the planet with the graph.

Enter the command **FnOn 4.5.6** to turn on the last three functions without disclosing their formulas in the Y= Editor.
(Use VARS Y-VARS 4:On/Off to paste the FnOn command on the home screen.)
Press DRAW 1:ClrDraw to refresh the movie and press GRAPH.

```
NORMAL FLOAT AUTO REAL RADIAN MP
FnOn 4,5,6
                                Done
ClrDraw
                                Done
■
```

Production Tip: If you select ClrDraw immediately from leaving the Graph Screen it will immediately redraw. If you select it from any other menu, the command is pasted on the Home Screen and you must then press GRAPH to refresh the screen.

Provide these three formulas: $a(t) = -1.86t^2 + 100$,

$$b(t) = -4.9t^2 + 100,$$
$$c(t) = -11.44t^2 + 100$$

Now ask students to match the formula to the planet and explain their reasoning. Students may connect the planet with the time it takes for the ball to hit the ground (which now you can ask them to find, if class time permits). Other students may point out the leading coefficient has vertically stretched or compressed the graph and affects the rate at which the ball is falling.
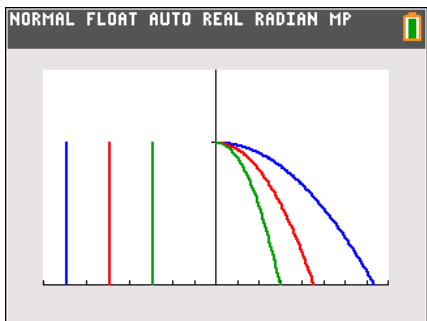
Yet even another approach: Instead of providing the formulas, ask students to construct a formula for $h(t)$, the height of the ball above ground in t seconds for each planet if $h(t) = \frac{1}{2} g t^2 + h_0$, where g is the acceleration due to gravity and h_0 is the initial height. For Mars, Earth, and Jupiter, $g = -3.72$, -9.8, and -22.88 meters per sec^2, respectively.

Home Run?

Adapted from a problem by Mike Koehler published in the Fall 1993 issue of Eightysomething!

The "Green Monster," the left field wall in Fenway Park in Boston is 37 feet high. Its distance from home plate is 379 feet. A baseball is hit by a Boston Red Sox player when it is 3 feet from the ground with an initial velocity of 123 feet per sec at an angle of elevation of 32°.

A wind of 3 mph (4.4 ft per sec) is blowing towards the batter. The ball is on a path that will take it to the deepest part of the Green Monster, 379 feet away. Is it a home run?

The initial force is resolved into horizontal (x) and vertical (y) components using trigonometry.

$$V_x = 123\cos 32°$$
$$V_y = 123\sin 32°$$
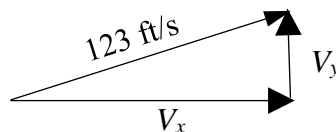
The horizontal parametric equation is $X(t) = V_x T + W$, where V_x is the horizontal component of the initial velocity, T is the time, and W is the other horizontal forces working, such as the wind.

The vertical parametric equation is $Y(t) = -16T^2 + V_y T + S_0$, where V_y is the vertical component of the initial velocity, T is the time, and S_0 is the starting height.
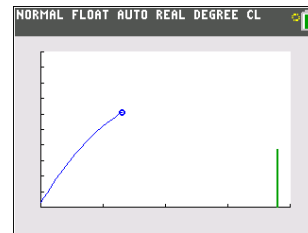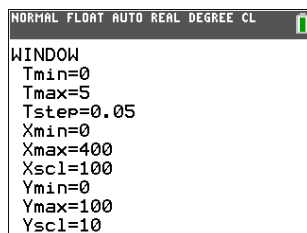
Enter the equations:

The ball: ■◦X₁ᴛ❏123cos(32°)T−4.4T
　　　　　　 Y₁ᴛ❏(⁻16T²+(123sin(32°)T+3)/(T<3.85)

The wall: ■\X₂ᴛ❏379
　　　　　　 Y₂ᴛ❏37T(T≤1)

The quantity (T<3.85) is 1 if true, 0 if false.
A division by 0 occurs for $t \geq 3.85$, so the domain is restricted to $t < 3.85$.

The quantity (T≤1) is 1 if true, 0 if false.
The wall will rise from the ground until, when $t = 1$, its height is 37 ft. and thereafter rise no more.
For $t > 1$ the height of the wall is 0.
The multiplication of (T≤1) creates a line connecting (379, 37) to (379, 0) when $t = 1$.

Press WINDOW. Use the settings shown.
Press GRAPH.

```
NORMAL FLOAT AUTO REAL DEGREE CL
WINDOW
 Tmin=0
 Tmax=5
 Tstep=0.05
 Xmin=0
 Xmax=400
 Xscl=100
 Ymin=0
 Ymax=100
 Yscl=10
```

The graph of Y₃ᴛ has a restricted domain of $t > 3.8$.   After playing the movie, you can see what it does!

■◦X₃ᴛ❏(758−X₁ᴛ)/(T≥3.80)
　Y₃ᴛ❏(⁻16T²+(123sin(32°)T+3)/(T≥3.80)

The graph of Y₄ᴛ is just for fun.
Position your cursor over the = sign,
press ENTER to highlight it,
and press GRAPH.

■\X₄ᴛ❏14cos((2880T)°)+{500,325,015,030,115,075}
　Y₄ᴛ❏7sin((2880T)°)+{88,85,85,70,87,72}

Store this into a GDB to recall later.

Roller Coaster

Animations can be done using Graph Styles and transformations of functions. The GraphStyle in Y6 creates a ball that has been raised to sit just above the "rollercoaster" sinusoidal path. The ball floats above the roller coaster track because the sine curve in Y3 has been translated vertically.

Note also that the sun and the bird are done with lists and restricted domains so that multiple transformations can be done as one graph. This gives the student the ability to graph more than 10 functions at a time.

```
NORMAL FLOAT AUTO REAL RADIAN CL

Plot1  Plot2  Plot3
■\Y₁█■({4,2,1..4}-(X+10)^2
)+5
■\Y₂█-√({4,2,1..4}-(X+10)^
2)+5
■\Y₃█2sin(.5X)-3
■\Y₄█(-8(X-8)^2+5)/((X≤8.4
 and X≥7.6))
■\Y₅█(-8(X-8.8)^2+5)/((X≥8
.4 and X≤9.2))
```

```
NORMAL FLOAT AUTO REAL RADIAN CL

WINDOW
 Xmin=-13.2
 Xmax=13.2
 Xscl=1
 Ymin=-8
 Ymax=8
 Yscl=1
 Xres=1
 ΔX=0.1
 TraceStep=0.2
```

```
NORMAL FLOAT AUTO REAL RADIAN CL

Plot1  Plot2  Plot3
■\Y₄█(-8(X-8)^2+5)/((X≤8.4
 and X≥7.6))
■\Y₅█(-8(X-8.8)^2+5)/((X≥8
.4 and X≤9.2))
■₀Y₆█2sin(.5X)-2.5
■\Y₇=
■\Y₈=
■\Y₉=
■\Y₀=
```
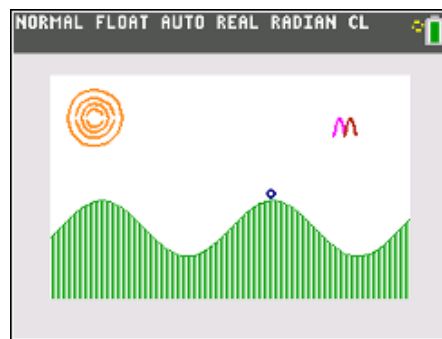
Math 99 Animated Movie

The program shown below was created for an Advanced Calculator Topics course (numbered MATH 99). It contains animated features such as fireworks and flashing lights to highlight the kind of CGI movies possible using a program.

These fireworks randomly appear on the screen:

After the title MATH 99 unveils,
the animation automatically repeats indefinitely.
Press the ON key to interrupt the program.
Then press **1:Quit**.
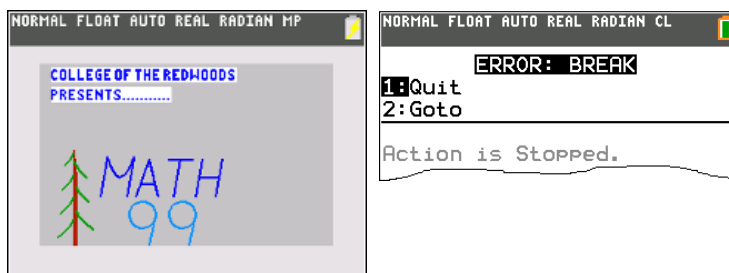
After running a program,
you may wish to Reset Defaults.
Press 2nd MEM 7 2 2.

Below is the code for the program MATH99.
It can be downloaded on the Website listed on the cover of this handout.

```
001  Radian
002  FnOff :PlotsOff :AxesOff
003  ClrDraw
004  BackgroundOn 22
005  0→Xmin
006  94→Xmax
007  0→Xscl
008  0→Ymin
009  62→Ymax
010  0→Yscl
011  Lbl 1
012
013  For(P,1,5,1)
014    ClrDraw
015    Text(2,10,"COLLEGE OF THE REDWOODS")
016    Text(20,10,"PRESENTS......")
017    randInt(35,60)→J
018    randInt(10,30)→K
019
020    For(Q,1,9,1)
021      randInt(10,21)→L
022      Line(J,K,randInt(30,65),randInt(5,45),1,L)
023    End
024
025  End
026  ClrDraw
027  Text(2,10,"COLLEGE OF THE REDWOODS")
028  Text(20,10,"PRESENTS...........")
029  Line(15,0,15,43,1,16,2)
030  Line(15.5,0,15.5,39,1,16,2)
031  Line(16,0,16,33,1,16,2)
032  DrawF e^(X/06)/((X≥08)(X≤15)),14
033  DrawF e^(X/06)/((X≥09)(X≤15))+10,14
034  DrawF e^(X/06)/((X≥10)(X≤15))+20,14
035  DrawF e^(X/06)/((X≥11)(X≤15))+30,14
036  DrawF e^((-X+30)/6)/((X≥16)(X≤24))+5,14
037  DrawF e^((-X+30)/6)/((X≥16)(X≤22))+16,14
038  DrawF e^((-X+30)/6)/((X≥15)(X≤21))+26,14
```

```
039  Line(30,39,27,22)
040  DrawF (.25(X-36)^2+31)/((X≥30)(X≤42))
041  Line(42,39,39,22)
042  Line(49,39,43,22)
043  Line(49,39,53,22)
044  Line(46,28,51,28)
045  Line(58,39,69,39)
046  Line(63,39,60,22)
047  Line(73,39,70,22)
048  Line(82,39,79,22)
049  Line(72,31,81,31)
050  For(I,1,02,1)
051    StorePic 7
052    ClrDraw
053    seq(X,X,1,001,1)→L₅
054    RecallPic 7
055    seq(X,X,1,001,1)→L₅
056  End
057  Circle(46,14,6,18)
058  Circle(64,14,6,18)
059  Line(52,14,49,0,1,18)
060  Line(70,14,67,0,1,18)
061  seq(X,X,1,475,1)→L₅
062  ClrDraw
063  Text(10,08,"ADV. CALCULATOR TOPICS")
064  Text(50,20,"WEDNESDAYS   1-3  PM"
065  Text(30,12,"STUART MOSKOWITZ")
066  DrawF (08sin(.2X)+15)/((X≥10)(X≤84))
067  seq(X,X,1,002,1)
068  Goto 1
```
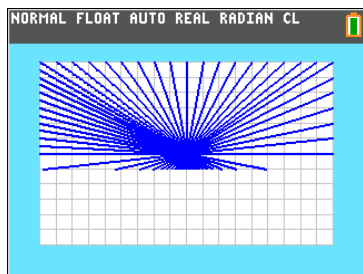
Spotlight Crawling on the Wall

A rotating beacon is 10 feet
from a long straight wall.
The beacon makes 1 revolution every 4 seconds.
The light hits the wall at a distance of d feet from the point P.
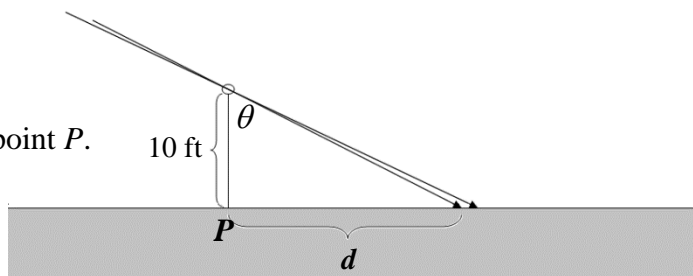
Write d as a function of θ.
Then write d as a function of t.

Transfer the program
BEACON.8XP to your
calculator to animate
the scenario.

To stop the program,
press the ON key.
Then press **1:Quit**.

BEACON.8XP

```
001   Radian:Param:FnOff :GridLine :BorderColor 3
002   .072→Tstep
003   -90→Xmin:90→Xmax:10→Xscl
004   -60→Ymin:60→Ymax:10→Yscl
005
006   Lbl 1
007   ClrDraw
008   For(T,-1+Tstep,1-Tstep,Tstep)
009   Line(0,0,10tan(πT/2),-10)
010   End
011   For(T,0,Ymax,10)
012   Line(0,0,Xmax,T)
013   End
014   For(T,Xmax,Xmin,-10)
015   Line(0,0,T,Ymax)
016   End
017   For(T,Ymax,0,-5)
018   Line(0,0,Xmin,T)
019   End
020   Goto 1
```

The program WALCRAWL.8XP shows the light crawling
across the wall in a continuous fashion. As before, press
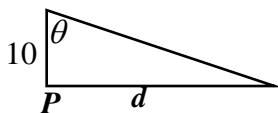the ON key to exit. Press 2nd MEM 7 2 2 to reset defaults.

WALCRAWL.8XP

```
001   Radian:Param:FnOff
002   "(10tan(πT/2))(-1<T)(T≤1){1,1,1,1,1,1,1,1,1,1}"→X₁ᴛ
003   "(-10)(-1<T)(T≤1){1,1,1,1,1,1,1,1,1,1}"→Y₁ᴛ
004   GraphStyle(1,5)
005   -1→Tmin:1→Tmax:.072→Tstep
006   -90→Xmin:90→Xmax:10→Xscl
007   -60→Ymin:60→Ymax:10→Yscl
008   Lbl 1
009   DispGraph
```

Using right triangle definitions, students find the formula of
d as a function of θ:

$$\tan\theta = \frac{d}{10} \Rightarrow d = 10\tan\theta$$

If negative values of d correspond to distances to the left of P,
and positive values of d correspond to distances to the right of P, students can explore symmetry.
From right triangle trigonometry, we can find values of d in the table.

As the value of $t \to \pm 1$, we have $\theta \to \pm\frac{\pi}{2}$ and $d \to \pm\infty$.

Since the beacon makes 1 revolution every 4 seconds, we have $\theta = \frac{\pi}{2}t$.

Students substitute $\theta = \frac{\pi}{2}t$ into $d = 10\tan\theta$ and so $d = 10\tan\left(\frac{\pi}{2}t\right)$.

t	θ	d
-1	$-\pi/2$	undefined
$-1/2$	$-\pi/4$	-10
0	0	0
$1/2$	$\pi/4$	10
1	$\pi/2$	undefined

Once you graph the function d, discuss with the class the connections
between graph and the behavior of the beam shown in the movie.

- At $t = 0$, the beam is exactly at the point P perpendicular to the wall. The distance d from P is zero.
  The graph of the tangent function passes through the origin.
- At $t = -0.5$ and $t = 0.5$ there is an isosceles right triangle ($\theta = \pm\frac{\pi}{4}$).

  The beam touches the wall at $d = 10$ ft from P.
- At $t = -1$ and $t = 1$ there is no triangle ($\theta = \pm\frac{\pi}{2}$). The beam does not touch the wall.

  The value of d is undefined. The graph of the tangent function has vertical asymptotes at $t = \pm 1$.
- When t is near 0, the beam is near the point P, and moves much more slowly, to a crawl.
  The graph of the tangent function does not have a steep slope near the origin (comparatively).
- When $t \to 1$, $\theta \to \frac{\pi}{2}$, and $d \to \infty$. The beam moves quickly as its distance on the wall from P climbs.
  The graph of the tangent function has a steep slope near the asymptote $t = 1$.
- Similarly, as $t \to -1$, $\theta \to -\frac{\pi}{2}$, and $d \to -\infty$ using symmetry.