Dig In to the Real World with the TI-84 Plus CE

27th Annual T^3 International Conference
Fort Worth, Texas

Friday, March 13, 2015
10:00 a.m. – 11:30 a.m.
Omni Hotel, Texas Ballroom H

John LaMaster, National T^3 Instructor
Indiana University Purdue University at Fort Wayne
2101 Coliseum Blvd. East
Kettler 264
Fort Wayne, IN 46805-1445
E-mail: lamaster@ipfw.edu
users.ipfw.edu/lamaster/technology/

Investigation 1: Ditch Diggers at http://threeacts.mrmeyer.com/ditchdiggers/

Prologue:

Common Core State Standards

8.EE.7: *Solve linear equations in one variable.*

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

MP4: *Model with Mathematics*

Act One:

Once you watch the video, what questions might arise from your students? Write in the space below

Will the two ditch diggers meet?
If they don't meet, what is the closest they will come to each other?
Are they digging at a steady rate?
If so how fast is each digging per day?
Are they digging at the *same* rate?
If not, which one is faster? How much faster?

Act Two: Pull from students suggestions on what information would be useful to know.

Coordinates are given for Digger 1 (X1, Y1) and Digger 2 (X2, Y2) for the first five days:

Day, *t*	X1	Y1	X2	Y2
0	0	2	68	34
1	2	3	67	33.5
2	4	4	66	33
3	6	5	65	32.5
4	8	6	64	32
5	10	7	63	31.5

The formula of the line for Y1(X1):

Slope: 0.5
y-intercept: (0, 2)

Formula: $Y1 = 0.5X1 + 2$

X1	Y1
0	2
2	3
4	4
6	5
7	6
10	7

The formula of the line for Y2(X2):

Slope: ________

Formula: Y2 = ___________

X2	Y2
68	34
67	33.5
66	33
65	32.5
64	32
63	31.5

Do the lines intersect?

Act Three: You can play the video at **http://threeacts.mrmeyer.com/ditchdiggers/** or, alternatively, use the TI-84 CE (or any in its family) to see the complete dig by modeling the data below.

X1(t) = ___________

Day, *t*	X1
0	0
1	2
2	4
3	6
4	8
5	10

Y1(t) = ___________

Day, *t*	Y1
0	2
1	3
2	4
3	5
4	6
5	7

.

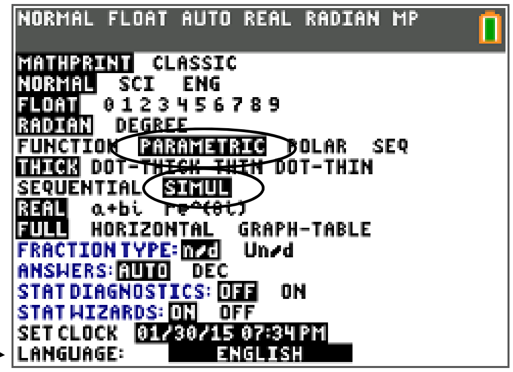
X2(t) = ___________

Day, *t*	X2
0	68
1	67
2	66
3	65
4	64
5	63

Y2(t) = ___________

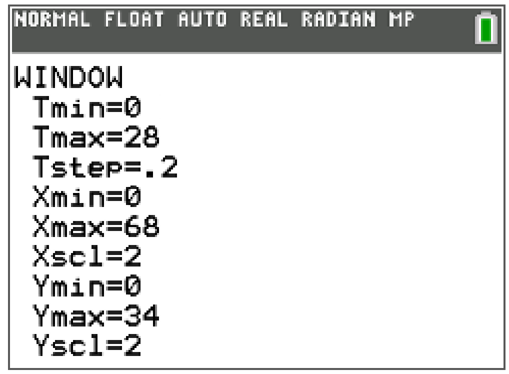
Day, *t*	Y2
0	34
1	33.5
2	33
3	32.5
4	32
5	31.5

1. Press MODE. Select **Parametric** and **Simul** graphing.

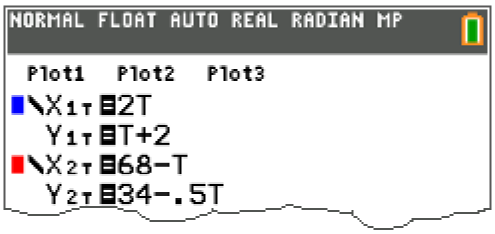


New: GO TO 2ND FORMAT GRAPH is replaced with
a language setting spinner

2. Use a window to dig for Tmax = 28 days.



3. Enter your equations in Y= and press GRAPH.

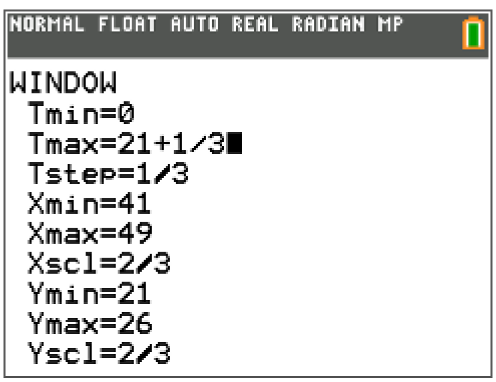


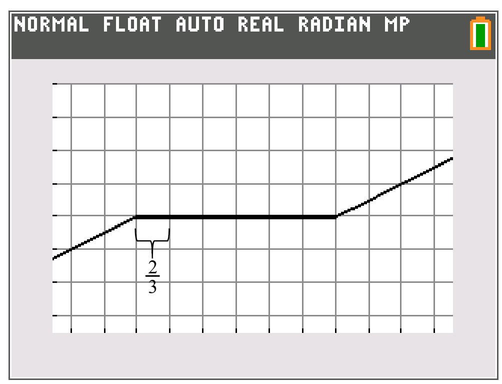
Sequel:

What is the closest they come to each other?
On what day should they stop digging in their original direction and pick a new one?

What if we set Y1=Y2?

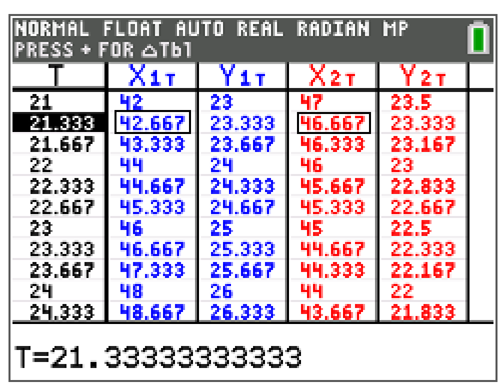
$$Y1 = Y2$$
$$t + 2 = 34 - \tfrac{1}{2}t$$
$$\tfrac{3}{2}t = 32$$
$$t = \tfrac{64}{3}$$
$$= 21\tfrac{1}{3}$$





Utilizing the grid, they come $6 \cdot \frac{2}{3} = 4$ units apart. (The line was added to the image using Paint.)

We could also use TRACE to find the coordinates
and then calculate the distance
Alternatively, use the table, i.e. 46.667 − 42.667 = 4.


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

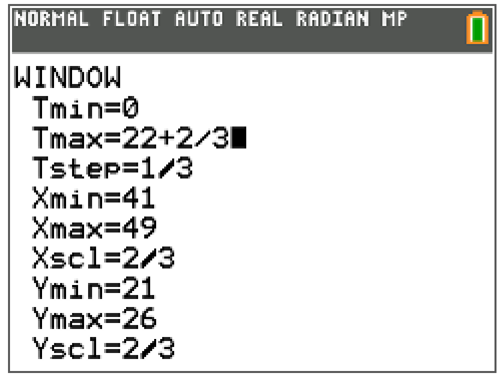
T	X1T	Y1T	X2T	Y2T
21	42	23	47	23.5
21.333	42.667	23.333	46.667	23.333
21.667	43.333	23.667	46.333	23.167
22	44	24	46	23
22.333	44.667	24.333	45.667	22.833
22.667	45.333	24.667	45.333	22.667
23	46	25	45	22.5
23.333	46.667	25.333	44.667	22.333
23.667	47.333	25.667	44.333	22.167
24	48	26	44	22
24.333	48.667	26.333	43.667	21.833

T=21.3333333333

Can we get closer?

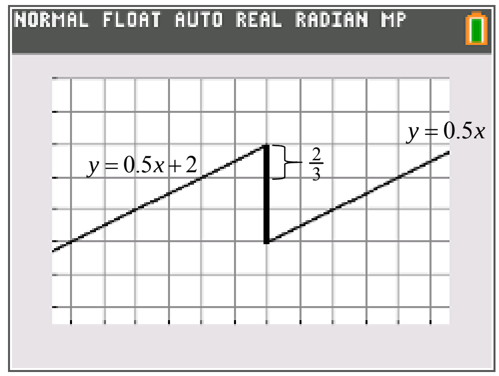
What if we set X1=X2?

$$X1 = X2$$
$$2t = 68 - t$$
$$3t = 68$$
$$t = \tfrac{68}{3}$$
$$= 22\tfrac{2}{3}$$

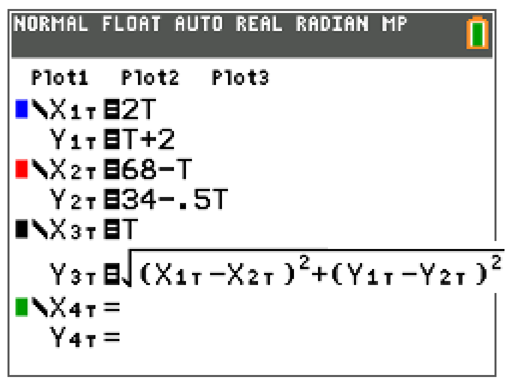


Utilizing the grid, they come $3 \cdot \frac{2}{3} = 2$ units apart.
We could also use the fact that
$y = 0.5x + 2$ is a vertical shift up 2 units of $y = 0.5x$.

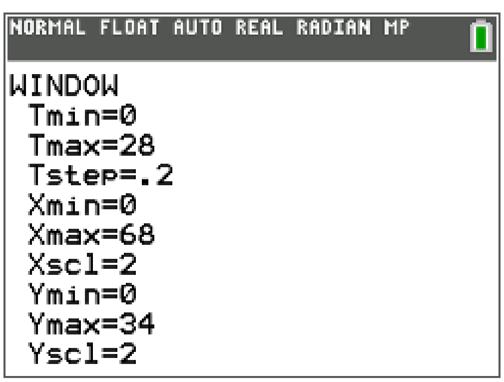
Can we come closer?

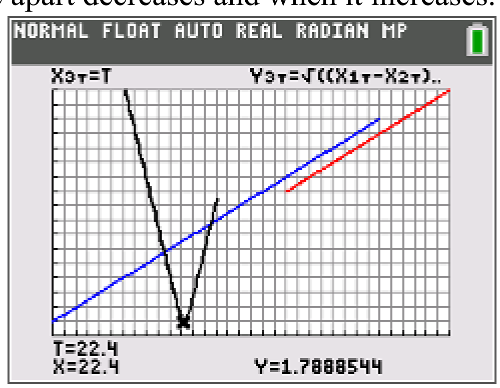


In Y3 we can create a function which gives the distance between the two diggers.



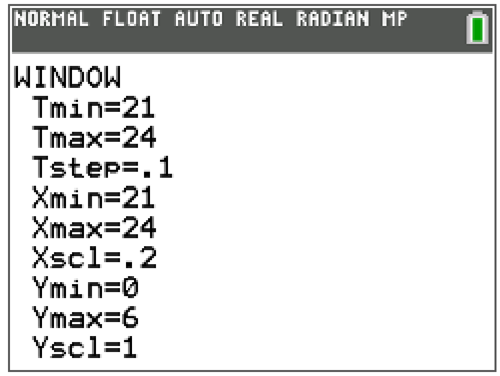
Use the previous window. Y3 shows when their distance apart decreases and when it increases.





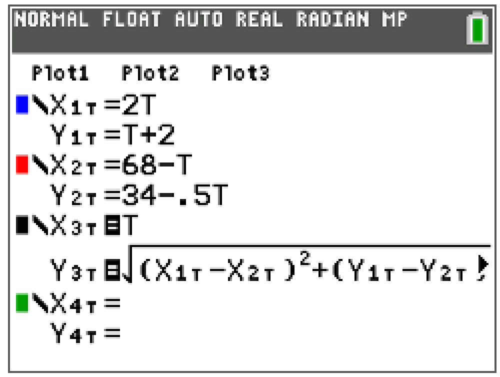
Press TRACE, type 22, and use down arrow to move to the Distance function.
Use the right arrow to find the minimum.

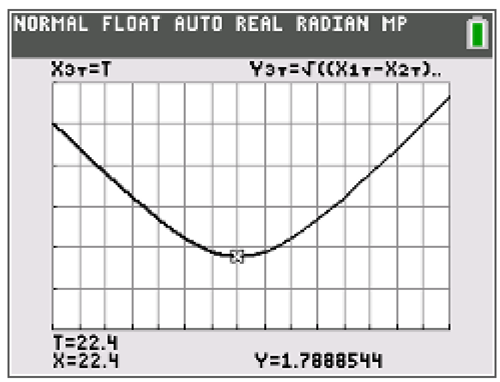
Set a new window or Zoom In to investigate the minimum.
The lower the value of Tstep, the smoother the curve.



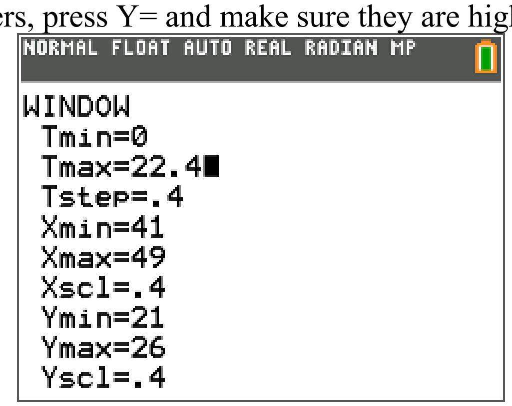
When moving from one curve to another the colored coordinates can help identify which curve you are tracing.

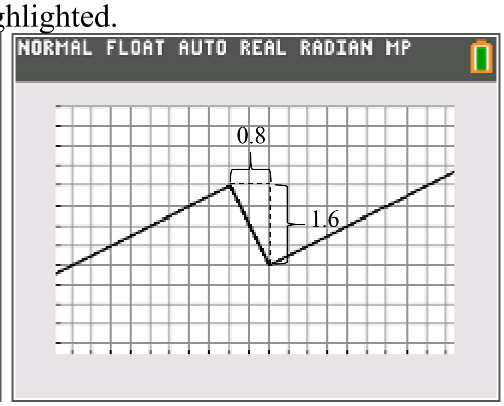
Optional Tip: You may want to deselect the diggers.
(In the Y= menu highlight the = sign and press ENTER.)





If you deselected the diggers, press Y= and make sure they are highlighted.





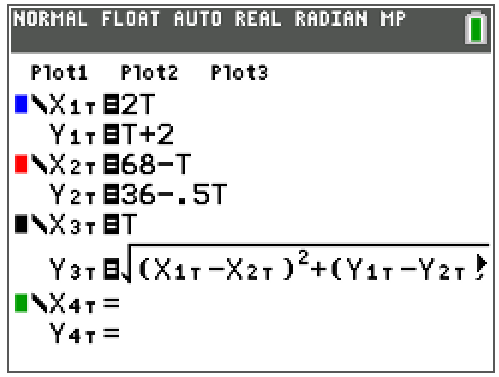
What does the grid reveal about the line connecting the diggers?
What is the exact value of the distance apart?

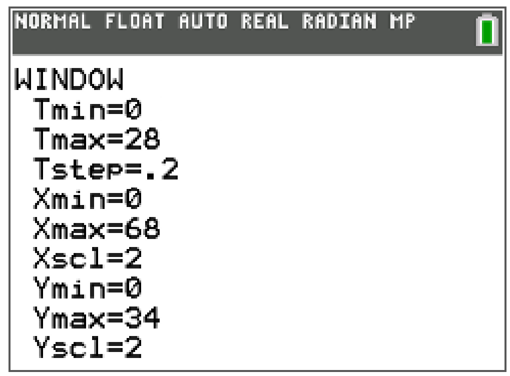
The connection line is perpendicular to the two parallel lines.
Using the Pythagorean Theorem we have the minimum distance is $\sqrt{(0.8)^2+(1.6)^2}=\sqrt{3.2}\approx 1.7885$.

Prequel (?)

What would be the model if the diggers actually met?
Shift the second digger up 2 units.

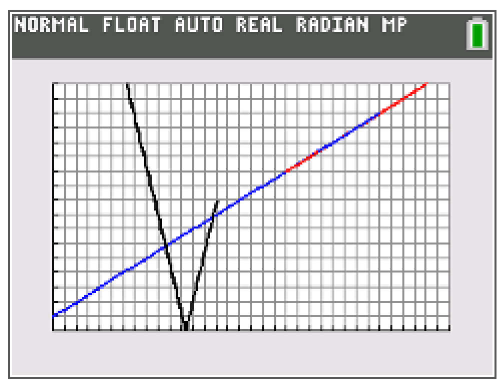
$$Y2=(34-.5t)+2$$
$$=36-0.5t$$





Set Y1=Y2 to find when they meet.

$$Y1=Y2$$
$$t+2=36-\tfrac{1}{2}t$$
$$\tfrac{3}{2}t=34$$
$$t=\tfrac{68}{3}$$
$$=22\tfrac{2}{3}$$



Notice the Distance Function touches the *x*-axis. Cool fact: Some algebra can show that the Distance Function in this case is a transformation of an absolute value function with zero at $t=\frac{68}{3}=22\frac{2}{3}$.

$$Y3=\sqrt{(X1-X2)^2+(Y1-Y2)^2}$$
$$=\sqrt{[(2t-(68-t)]^2+[(t+2)-(36-\tfrac{1}{2}t)]^2}$$
$$=\sqrt{(2t-68+t)^2+(t+2-36+\tfrac{1}{2}t)^2}$$
$$=\sqrt{(3t-68)^2+(\tfrac{3}{2}t-34)^2}$$
$$=\sqrt{(3t-68)^2+(\tfrac{1}{2}\cdot 2(\tfrac{3}{2}t-34))^2}$$
$$=\sqrt{(3t-68)^2+(\tfrac{1}{2}\cdot(3t-68))^2}$$
$$=\sqrt{(3t-68)^2+\tfrac{1}{4}\cdot(3t-68)^2}$$
$$=\sqrt{\tfrac{5}{4}(3t-68)^2}$$
$$=\tfrac{\sqrt{5}}{2}\left|3t-68\right|$$

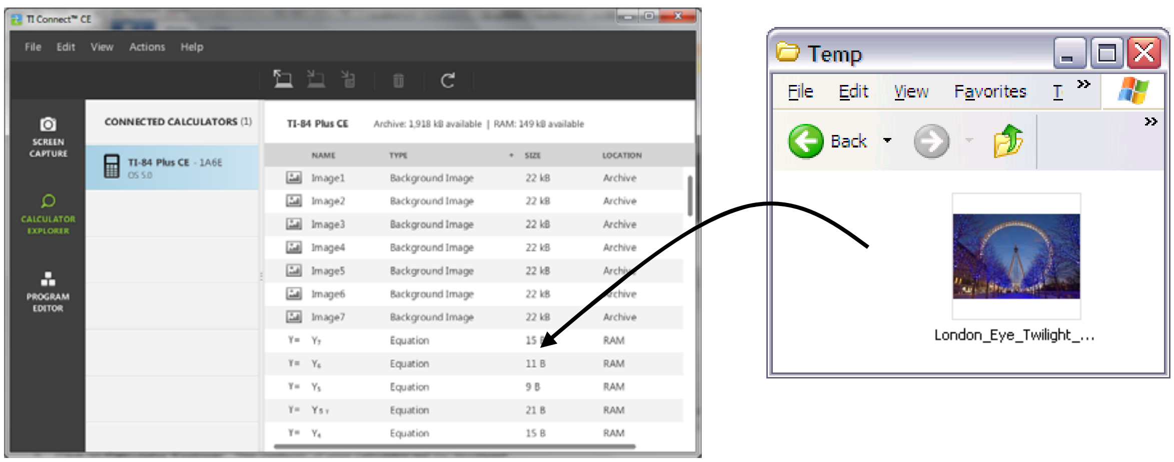
Investigation 2: Ride the London Eye

We will use a photo from the Wikipedia. Commons.
Permission is granted to copy, distribute and/or modify this image.
You can download the image from or my Website (see cover).
http://en.wikipedia.org/wiki/File: London_Eye_Twilight_April_2006.jpg

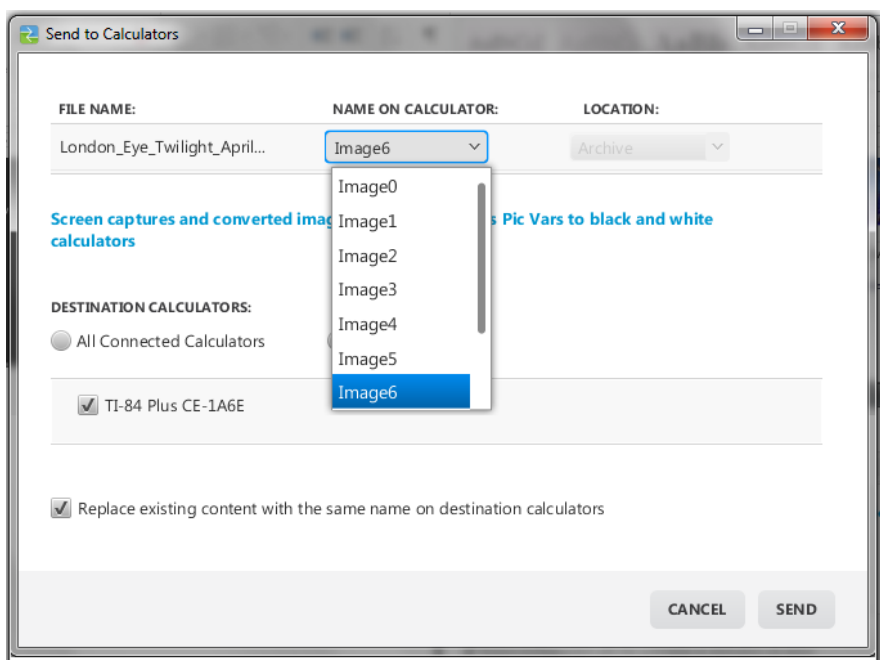
You will need the black link cable
to move the image to your calculator.
TI Connect™ 5.0 does not support the silver cable.

You can get TI Connect™ 5.0 from the accessories tab at education.ti.com/84ce.

1. Start the TI Connect™ software that has already been installed on your machine.
2. Connect the TI-84Plus CE to the computer with the black cable.
3. Click on **Calculator Explorer** . The contents of your calculator will be displayed.
4. Drag the digital image over. It will convert it into a Background Image file and prompt where to put it.



It will convert it into a Background Image file, prompt you for a location, and send it to your calculator.

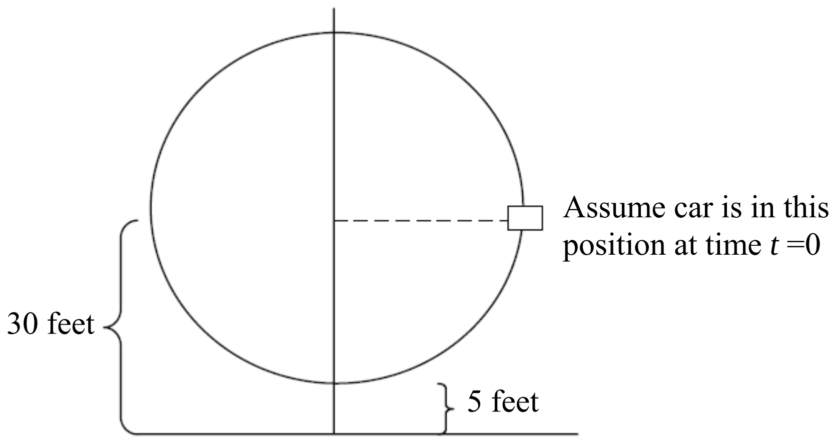


Similarly, send the program FERRIS.8xp to your calculator, available from my Website.
You now are riding the London Eye.

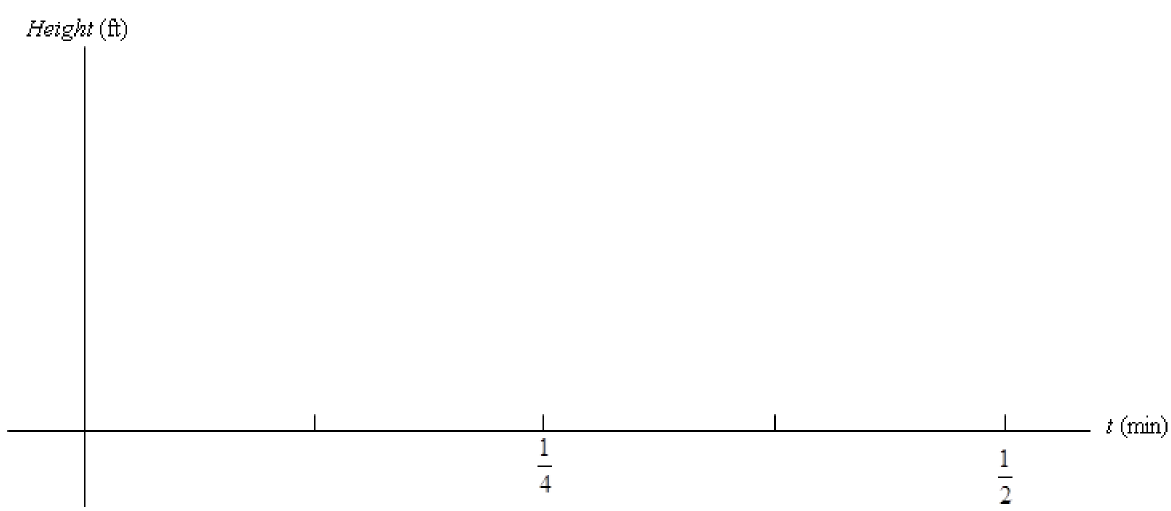
Ask students to compare the both paths of the height of the car.
When is the rate of change of the vertical height of the car with respect to time the fastest?

Classroom use: While students the "TI-84CE movie" is playing students work in groups on the activity below (which involves a different Ferris Wheel than the London Eye).

A Ferris Wheel 50 ft in diameter makes 4 revolutions in one minute.
The center of the wheel is 30 ft. above the ground. Assume the car travels **counterclockwise**.

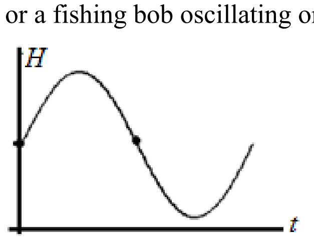


Without using an equation,
graph on paper the height of the Ferris Wheel car, in feet, for a half minute ride.
At t = 0, assume the car is in the position shown above (in the 3 o'clock position).



Use TRACE on the vertical line to explore the speed of the car.
(Compare this motion to a vibrating spring or a fishing bob oscillating on a lake.)

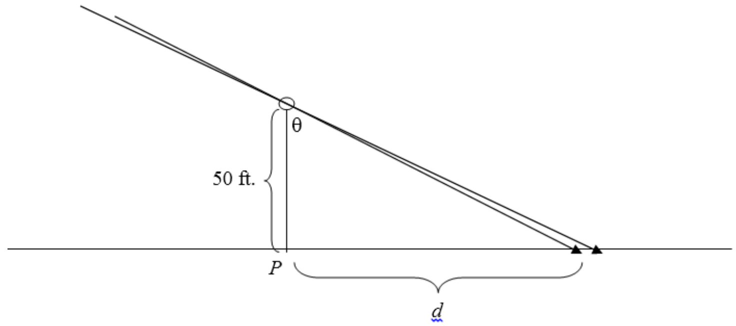
The car is fastest at the
3 o'clock and 9 o'clock positions,
where the curve has
the steepest slope.



Investigation 3: Spotlight

A spotlight is 50 ft. from a wall.
The light hits the wall at a distance of *d* feet
from the point *P*.

The spotlight makes 1 revolution every 2 seconds.
Adapted from Larson *Precalculus*: A Graphing Approach.



Transfer the program SPTLGHT.8xp to your calculator to animate the scenario.
(The program ONMYWALL.8xp shows the light moving across the wall in a continuous fashion.)

When done, press the ON key to interrupt the program.
Then press **1:Quit**.

While this movie is playing,
it is natural to wonder about
the rate of change of *d* with respect to time.

Using the above right triangle,
students find the formula of *d* as a function of θ:

$$\tan\theta = \frac{d}{50}$$
$$d = 50\tan\theta$$

If negative values of *d* correspond to distances to the left of *P*, and positive values of *d* correspond to distances to the right of *P*, students can explore symmetry.

Students find values of *d* at special values, i.e., $\theta = 0$, $\theta = \frac{\pi}{4}$, $\theta = -\frac{\pi}{4}$, $\theta = \frac{\pi}{2}$, $\theta = -\frac{\pi}{2}$ and as $\theta \to \pm\frac{\pi}{2}$.

If the spotlight makes 1 revolution every 2 seconds, students can write θ as a function of *t*: $\theta = \pi t$.
Students substitute $\theta = \pi t$ into $d = 50\tan\theta$ to create the formula $d = 50\tan \pi t$.

Once you graph the function *d*, discuss the relationship between the slope of the curve and the rate of change of the distance *d* with respect to time.