# *"How-To"*

# With Your TI-84 Plus CE Graphing Calculator Part 3

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This handout and related accessories (programs, image files, graphical databases) can be downloaded at <u>http://users.pfw.edu/lamaster/technology</u> if you scroll to the presentation *Create Silent Movies to Model the Real World*, 29th Annual T<sup>3</sup> International Conference, Chicago, IL, March 11, 2017. On this page you can also get links to Part 1 and 2 of this Webinar Series.

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# Lights! Camera! Action! – Three Ways to Produce Movies on the TI-84 Plus Family Calculator

#### 1. Live Action

Using parametric mode, many classical story problems from traditional algebra and precalculus textbooks can be brought to life by entering the settings in four menus: Mode, Format, Y=, and Window. In the activities *Harry and Sally* and *Batman and the Joker*, the teacher directs the activity from the display calculator live before the class (with some prior setup). Students mathematize the problem to tell the teacher what value of Tmax will stop the movie to show when the main characters meet. The activity *Ditch Diggers* is similar. The student activity *Alexander Hamilton and Aaron Burr* is designed so the settings could be entered by the students to create the movie.

#### 2. Function Transformations and Graph Styles saved with a Graphical Database (GDB)

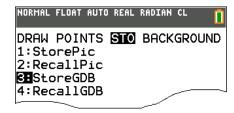
A graph database (GDB) stores the set of elements that defines a particular graph. You can recreate the graph from these elements. You can store up to 10 GDBs in variables GDB0 through GDB9 and recall them to recreate graphs.

A GDB stores five elements of a graph

- Graphing mode
- Window variables
- Format settings
- All functions in the Y= editor and the selection status of each
- Graph style for each Y= function

GDBs do not contain drawn items (created from the Draw Menu) or stat plot definitions.

Use 2nd Draw and press the right arrow to the STO menu to enter the command **StoreGDB 1** on the Home Screen to store GDB 1. To recall it, enter the command **RecallGDB 1**.



Some disadvantages with GDB's:

- Limited to 10 GDB's. However, you can store them in a group file, and there is no limit to the number of group files you may have (up to calculator memory).
- It is easy to accidentally overwrite a GDB by storing a new one over an existing one.
- Cannot be given descriptive names so it is difficult to know what is in it. TIP: When you store them onto a computer, you can rename the file.
  - You can assign a name to each GDB and then create a program to index them.
- While GDBs store most Mode and Format settings, it does not store Degree vs Radian Mode. Some examples using GDB's are *Bus Chase*, *Falling Objects on Different Planets*, *Home Run?* and *Roller Coaster*.

#### 3. Use a program

There is no limit to the number of programs you may have (up to calculator memory) and these can be given somewhat descriptive names. Much more control is possible, in particular, setting mode to Degree or Radians, computer generated imagery (CGI) such as random animated fireworks and flashing lights (*Math 99 Animated Movie*) or replaying automatically using an infinite loop (*Math 99 Animated Movie* and *Spotlight Crawling on a Wall*).

#### Harry and Sally

Adapted from an Algebra 1 Textbook: Harry the Hare and Sally the Tortoise have a race. Sally travels at a slow but steady speed of 1.6 mph. Harry travels at constant speed of 40 mph, but gives Sally a 12 hour head start. If the finish line is 21 miles away, who will win the race?

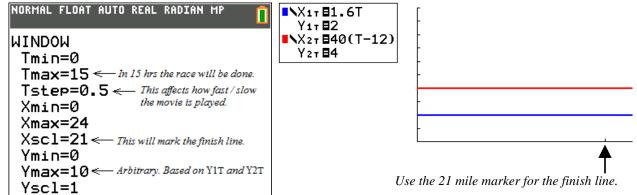
We will make a silent movie of this scenario to bring this iconic fable to life. Let Harry travel along the line y = 4 and Sally travel along the line y = 2.

1. Press MODE.

Highlight parametric mode and simultaneous graphing.



2. Press WINDOW and enter these settings. Press Y= and enter these equations. Press GRAPH.



#### A Look Behind the Scenes: What is happening?

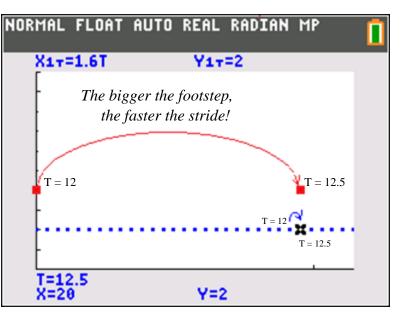
Press MODE, highlight DOT-THICK.

Press GRAPH. Press TRACE and >. Type 12 to jump to T= 12 hours.

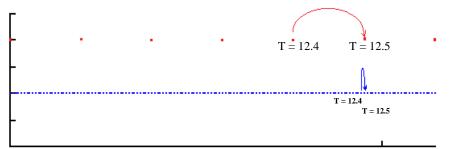
Press ; to move between graphs. In THICK (or CONNECTED) Graphing, the TI-84 connects these dots with lines.

Footsteps for each graph are connected simultaneously, so Harry will graph remarkably faster than Sally.

Lowering Tstep adds more footprints. This will slow down the action.



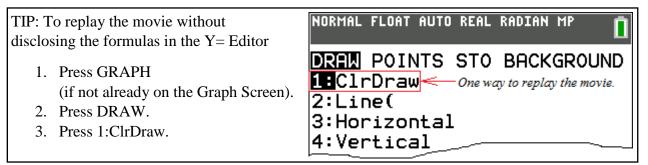
In Dot mode, TRACE from T = 12 to T = 12.5 with TStep = 0.5. Compare the leaps between footprints for the Hare and the Tortoise. Set Tstep = 0.1. The distance between footprints is smaller and more footprints appear.



In Dot mode, TRACE from T = 12.4 to T = 12.5 in steps of TStep = 0.1.

#### **Class Activity: Use the movie to heighten student engagement**

Set up your calculator as in Step 2 (with THICK or CONNECTED and Tstep = 0.5). Present the problem to students. Display to the class the movie on your calculator. (Students are not using their calculator in this activity to create the movie.).



- Who won? Set Tstep = 0.05 to slow down the movie. Press GRAPH.
- Find when they meet, i.e., *When Harry met Sally* <sup>(2)</sup>
   Replay and ask the class to tell you when to pause the action when they meet (by pressing ENTER). (This usually is not easy without Jedi skills.)

Easier: Have the class write formulas for each racer and solve equations algebraically:

1.6t = 40(t - 12) 1.6t = 40t - 480 38.4t = 480 $t = \frac{480}{38.4} = 12.5$ 

Assign Tmax = 12.5, Tstep = 0.5, and press GRAPH.

- *Optional:* ask the class to generate additional questions.
  - At what mile marker did they meet? Answer: 20 miles

- NORMAL FLOAT AUTO REAL RADIAN MP
- How long did it take each to cross the finish line?
   Answer: Sally took 13.125 hr =13 hr, 7.5 min; Harry took 12.525 hr = 12 hr, 31.5 min
- What are the values of *t* for which Harry is in the lead? How many seconds is this? Answer: From t = 12.5 to t = 12.525. This was the last 0.025 hr = 1.5 min = 90 seconds of the race!

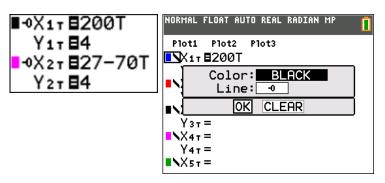
#### Batman and the Joker

This problem is similar to the Harry and Sally problem in that you can direct it from the Teacher Calculator while engaging students in the discussion.

Batman is traveling 200 mph due East when he sees the Joker coming directly toward him 27 miles away, traveling at 70 mph West. How long until they meet?

- Set up your TI-84CE. Highlight parametric mode and simultaneous mode.
- 2. Press Y= and enter these equations:

Optional: Use the spinner to change graph color (Black/Magenta) and graph style (Path).



3. <u>Set the window.</u>

Tmin=0
<b>Tmax=0.2</b> (Set Tmax to be the length of time, in hours, you want to see them driving)
<b>Tstep=0.005</b> (adjust Tstep to taste; the higher the Tstep, the faster the movie.)
Xmin=0
Xma = 27 (set Xmax to be the initial distance they are apart.)
Xscl=1
Ymin=0
Ymax=10
Yscl=1

Change the above movie to play for Tmax = 0.05 hours (*or consider using 60ths of an hour*) Show the class that the movie stopped too soon. Bummer.

- 4. Now ask, "When shall I stop the clock so that the movie ends **exactly** when they meet?" Students set up the equation 200t = 27-70t and solve for *t* algebraically.
- 5. Once the solution t = 0.1 is found, change Tmax = 0.1. Press GRAPH to replay.

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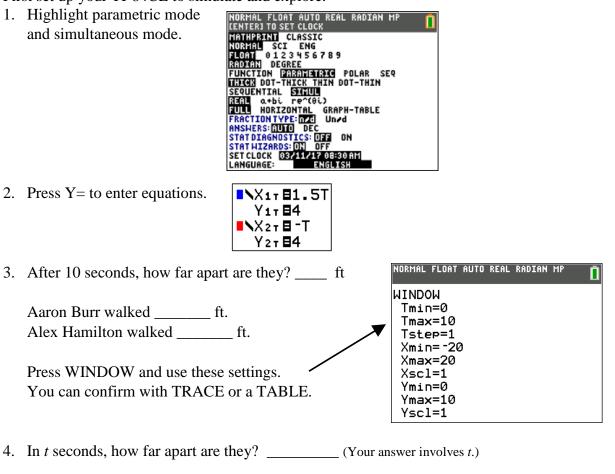
Here's another one, similar to the above:

Batman's speed is 185 mph, Joker's speed is 55 mph and they are initially 60 miles apart.

#### **Alexander Hamilton and Aaron Burr**

Alex and Aaron meet on the dueling ground, each holding a Super Soaker water pistol. They stand back to back and simultaneously walk until they are 20 ft apart. Alex walks 1 ft per sec West, while Aaron (a bit jittery) walks 1.5 ft per sec East. How many seconds before the distance between them is 20 ft?

First set up your TI-84CE to simulate and explore.



Aaron Burr walked \_\_\_\_\_ ft. (Your answer involves t.)Alex Hamilton walked \_\_\_\_\_ ft. (Your answer involves t.)

5. Write an equation to determine the value of *t* which makes their distance apart equal to 20 ft. Solve the equation. Set Tmax equal to this value of *t* and press GRAPH.

Complete: They are 20 ft apart in  $t = \_$  seconds; Aaron Burr walked \_\_\_\_\_ ft and Alex Hamilton walked \_\_\_\_\_ ft

#### **Ditch Diggers**

Dan Meyer incorporates video at <u>http://threeacts.mrmeyer</u> to teach with three-act math tasks. (See <u>http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/</u> for his approach.)

#### Act One:

Introduce the story/task using as few words as possible.

In this example: Two ditch diggers start digging towards each other from opposite directions.

The three acts for this movie are available at <u>http://threeacts.mrmeyer.com/ditchdiggers/</u>; however, this movie can be simulated on the TI-84 Family calculator.

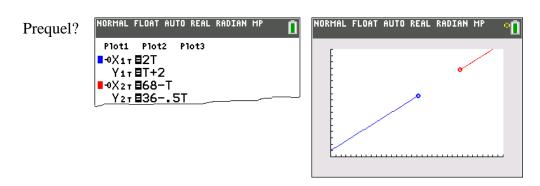


See *Dig In to the Real World with the TI-84 Plus CE* at <u>http://users.ipfw.edu/lamaster/technology/</u> from the 27th Annual T<sup>3</sup> International Conference for details on how to incorporate the calculator into this activity.

Act Two: Coordinates are given for Digger 1 ( $X_{1T}$ ,  $Y_{1T}$ ) and Digger 2 ( $X_{2T}$ ,  $Y_{2T}$ ) for the first five days to develop the following formulas:

NORMAL FLOAT AUTO REAL RADIAN MP
Plot1 Plot2 Plot3
NX1T ■2T Y1T ■T+2
■X27 ■68-T Y27 ■345T
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW Tmin=0
Tmax=28 Tstep=.2
Xmin=0
Xmax=68 Xscl=2
Ymin=0 Ymax=34
Ymax=34 Ysc1=2

Act Three: Complete the task. Set up a sequel/extension. Spoiler: The diggers do not meet.



# **Bus Chase**

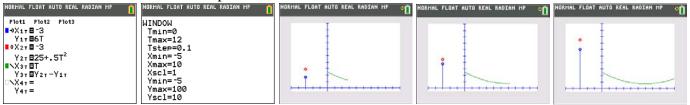
This problem was published in COMAP Consortium Number 42 in Summer 1992.

A man who is late for work is approaching a bus. When he is 25 meters from the bus it starts to pull away with an acceleration of 1.0 meter per second per second. He sees this and begins to run at 6 meters per second. Will he catch the bus? If not, how close will he come?

First we will represent the position of the man and the bus (relative to the man's starting point) on vertical lines on the screen. After *t* seconds the man's distance from his starting point is 6t and the bus's distance from the man's starting point is  $25 + 0.5t^2$ .

- 1. Use  $X_{1T}$  and  $Y_{1T}$  to represent the man's motion and  $X_{2T}$  and  $Y_{2T}$  to represent the bus's motion. Make the man and the bus move on the vertical line x = -3. Set the equations and the window to do this. Use SIMUL mode. Use O Line Style for the bus and dotted line style for the man.
- 2. Trace to see if he catches the bus.
- 3. Graph the distance between the bus and the man versus time *t*. To do this let  $X_{3T} = T$  and  $Y_{3T} = Y_{1T} Y_{2T}$ . Keep the window set so that you can see the vertical motions and the plot of  $Y_{3T}$ .

Screenshots from the **Bus Chase** problem:



# **Falling Objects on Different Planets**

Teacher: "Three balls will be dropped from a height of 100 meters on each of the planets, Mars, Earth, and Jupiter."

Show the TI-84 movie of the balls dropping. Do not show formulas yet. "Discuss in your groups:

What is similar about each? What is different?

Which planet goes with which graph?"

Prior to class set up the calculator using the screens to the right. A GDB is an excellent way to have this ready to go for class.

Students may bring up the following similarities:

- Each has the same initial height.
- As they fall, they move faster and faster.

Students may bring up the following differences:

- Length of time to drop.
- Rate of acceleration.

#### Match:

The slowest is Jupiter, fastest is Mars, and Earth is in between. The larger the planet's mass, the larger the acceleration due to gravity. Plot1 Plot2 Plot3  $X_{1T} \equiv 1$   $Y_{1T} \equiv -1.86T^{2}+100$   $X_{2T} \equiv 3$   $Y_{2T} \equiv -4.9T^{2}+100$   $X_{3T} \equiv 5$   $Y_{3T} \equiv -11.44T^{2}+100$   $X_{4T} =$  $Y_{4T} =$ 

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Xmax	• -					
Xsc]						
Ymir Vmes	ש=ט <=150	2				
	(=100					
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# Falling Objects on Different Planets (Alternative)

Based on students' level, interest, or available class time, the previous activity can be extended. Instead of setting up the calculator with only the three balls dropping, use the screens to the right, where the last three graphs are deselected (not highlighted) and use the more expanded viewing window  $-8 \le x \le 8$ .

As before, play the movie while asking students to find similarities, differences, and to match the planet with the graph.

Enter the command **FnOn 4.5.6** to turn on the last three functions without disclosing their formulas in the Y= Editor. (Use VARS Y-VARS 4:On/Off to paste the FnOn command on the home screen.) Press DRAW 1:ClrDraw to refresh the movie and press GRAPH.

**Production Tip:** If you select ClrDraw immediately from leaving the Graph Screen it will immediately redraw. If you select it from any other menu, the command is pasted on the Home Screen and you must then press GRAPH to refresh the screen.

Provide these three formulas:  $a(t) = -1.86t^2 + 100$ ,

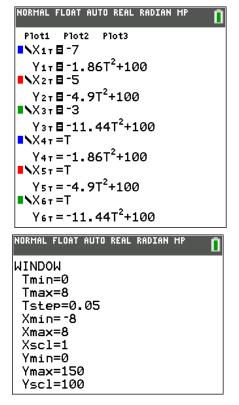
 $b(t) = -4.9t^2 + 100,$ 

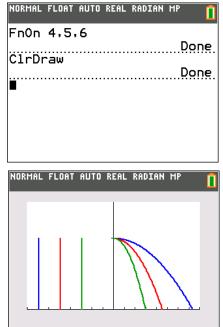
$$c(t) = -11.44t^2 + 100$$

Now ask students to match the formula to the planet and explain their reasoning. Students may connect the planet with the time it takes for the ball to hit the ground (which now you can ask them to find, if class time permits). Other students may point out the leading coefficient has

vertically stretched or compressed the graph and affects the rate at which the ball is falling.

Yet even another approach: Instead of providing the formulas, ask students to construct a formula for h(t), the height of the ball above ground in *t* seconds for each planet if  $h(t) = \frac{1}{2}gt^2 + h_0$ , where *g* is the acceleration due to gravity and  $h_0$  is the initial height. For Mars, Earth, and Jupiter, g = -3.72, -9.8, and -22.88 meters per sec<sup>2</sup>, respectively.







#### Home Run?

Adapted from a problem by Mike Koehler published in the Fall 1993 issue of Eightysomething!

The "Green Monster," the left field wall in Fenway Park in Boston is 37 feet high. Its distance from home plate is 379 feet. A baseball is hit by a Boston Red Sox player when it is 3 feet from the ground with an initial velocity of 123 feet per sec at an angle of elevation of 32°.

A wind of 3 mph (4.4 ft per sec) is blowing towards the batter. The ball is on a path that will take it to the deepest part of the Green Monster, 379 feet away. Is it a home run?

The initial force is resolved into<br/>horizontal (x) and vertical (y)<br/>components using trigonometry. $V_x = 123\cos 32^\circ$ <br/> $V_y = 123\sin 32^\circ$ 123 ft/s $V_x$ 

The horizontal parametric equation is  $X(t) = V_x T + W$ , where  $V_x$  is the horizontal component of the initial velocity, *T* is the time, and *W* is the other horizontal forces working, such as the wind. The vertical parametric equation is  $Y(t) = -16T^2 + V_y T + S_0$ , where  $V_y$  is the vertical component of the

initial velocity, T is the time, and  $S_0$  is the starting height.

Enter the equations:

The ball: ■•X1T ■123cos(32°)T-4.4T Y1T ■(-16T<sup>2</sup>+(123sin(32°)T+3)/(T<3.85)

 The quantity (T<3.85) is 1 if true, 0 if false.

A division by 0 occurs for  $t \ge 3.85$ , so the domain is restricted to t < 3.85.

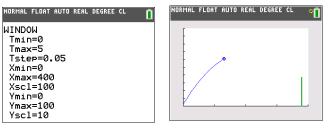
 $V_{v}$ 

The quantity  $(T \le 1)$  is 1 if true, 0 if false.

The wall will rise from the ground until, when t = 1, its height is 37 ft. and thereafter rise no more. For t > 1 the height of the wall is 0.

The multiplication of (T≤1) creates a line connecting (379, 37) to (379, 0) when t = 1.

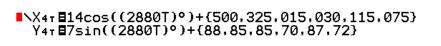
Press WINDOW. Use the settings shown. Press GRAPH.



The graph of  $Y_{3T}$  has a restricted domain of t > 3.8. After playing the movie, you can see what it does!  $-0X_{3T} \equiv (758 - X_{1T}) / (T \ge 3.80)$ 

Y3T ∎(-16T2+(123sin(32°)T+3)/(T≥3.80)

The graph of  $Y_{4T}$  is just for fun. Position your cursor over the = sign, press ENTER to highlight it, and press GRAPH.

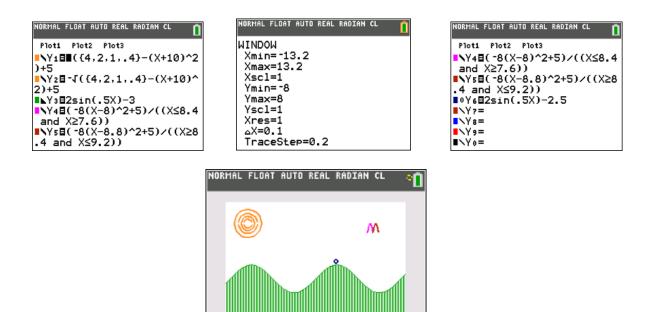


Store this into a GDB to recall later.

#### **Roller Coaster**

Animations can be done using Graph Styles and transformations of functions. The GraphStyle in Y6 creates a ball that has been raised to sit just above the "rollercoaster" sinusoidal path. The ball floats above the roller coaster track because the sine curve in Y3 has been translated vertically.

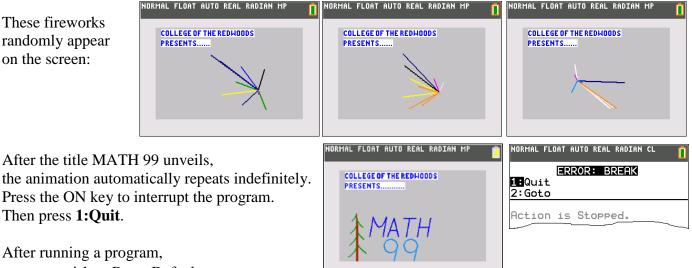
Note also that the sun and the bird are done with lists and restricted domains so that multiple transformations can be done as one graph. This gives the student the ability to graph more than 10 functions at a time.



#### Math 99 Animated Movie

The program shown below was created for an Advanced Calculator Topics course (numbered MATH 99). It contains animated features such as fireworks and flashing lights to highlight the kind of CGI movies possible using a program.

These fireworks randomly appear on the screen:



After running a program, you may wish to Reset Defaults. Press 2nd MEM 7 2 2.

Below is the code for the program MATH99.

Ø37 DrawF e^((-X+3Ø)/6)/((X≥16)(X≤22))+16,14 Ø38 DrawF e^((-X+3Ø)/6)/((X≥15)(X≤21))+26,14

It can be downloaded on the Website listed on the cover of this handout.

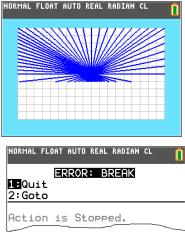
```
ØØ1
     Radian
                                                                 Line (30, 39, 27, 22)
                                                            039
     Fn0ff :Plots0ff :Axes0ff
                                                                 DrawF (.25(X-36)<sup>^</sup>2+31)/((X≥3Ø)(X≤42))
ØØ2
                                                            Ø4Ø
ØØ3 ClrDraw
                                                            Ø41
                                                                 Line (42, 39, 39, 22)
ØØ4
     BackgroundOn 22
                                                            042
                                                                 Line (49, 39, 43, 22)
005
     Ø→Xmin
                                                            Ø43 Line (49, 39, 53, 22)
ØØ6 94→Xmax
                                                            Ø44 Line (46, 28, 51, 28)
ØØ7
     Ø→Xsc1
                                                            Ø45 Line (58,39,69,39)
008
     Ø→Ymin
                                                            046
                                                                 Line (63, 39, 60, 22)
ØØ9 62→Ymax
                                                            Ø47
                                                                 Line (73, 39, 70, 22)
                                                            Ø48 Line (82, 39, 79, 22)
Ø1Ø Ø→Yscl
Ø11 Lb1 1
                                                            Ø49 Line (72,31,81,31)
Ø12
                                                            Ø5Ø
                                                                 For (I, 1, Ø2, 1)
Ø13 For (P, 1, 5, 1)
                                                            Ø51
                                                                   StorePic 7
Ø14
      C1rDraw
                                                            Ø52
                                                                   C1rDraw
Ø15
      Text(2,10, "COLLEGE OF THE REDWOODS")
                                                                   seq(X,X,1,ØØ1,1)→L 5
                                                            Ø53
Ø16
                                                            Ø54
      Text (20, 10, "PRESENTS.....")
                                                                   RecallPic 7
Ø17
                                                            Ø55
                                                                   seq(X,X,1,ØØ1,1)→L 5
      randInt(35,6Ø)→J
Ø18
                                                            Ø56 End
      randInt(1Ø,3Ø)→K
Ø19
                                                            Ø57
                                                                 Circle(46,14,6,18)
       For (Q, 1, 9, 1)
Ø2Ø
                                                                 Circle(64,14,6,18)
                                                            Ø58
        randInt(1Ø,21)→L
Ø21
                                                            Ø59
                                                                 Line (52, 14, 49, Ø, 1, 18)
        Line (J,K, randInt (30,65), randInt (5,45), 1,L)
Ø22
                                                            Ø6Ø
                                                                 Line (70, 14, 67, 0, 1, 18)
       End
Ø23
                                                            Ø61
                                                                 seq(X,X,1,475,1)→L5
Ø24
                                                            Ø62
                                                                 C1rDraw
Ø25 End
                                                                 Text(1Ø,Ø8,"ADV. CALCULATOR TOPICS")
                                                            Ø63
Ø26 ClrDraw
                                                            Ø64
                                                                 Text (50, 20, "WEDNESDAYS
                                                                                            1-3 PM"
Ø27
     Text(2,10, "COLLEGE OF THE REDWOODS")
                                                            Ø65
                                                                 Text (30, 12, "STUART MOSKOWITZ")
Ø28 | Text (2Ø,1Ø, "PRESENTS.....")
                                                            Ø66
                                                                 DrawF (Ø8sin(.2X)+15)/((X≥1Ø)(X≤84))
Ø29 Line (15, Ø, 15, 43, 1, 16, 2)
                                                                 seq(X, X, 1, \emptyset \emptyset 2, 1)
                                                            Ø67
Ø3Ø Line(15.5,Ø,15.5,39,1,16,2)
                                                            Ø68 Goto 1
Ø31 Line (16, Ø, 16, 33, 1, 16, 2)
Ø32 DrawF e^(X/Ø6)/((X≥Ø8)(X≤15)),14
Ø33 DrawF e^(X/Ø6)/((X≥Ø9)(X≤15))+1Ø,14
Ø34 DrawF e^(X/Ø6)/((X≥1Ø)(X≤15))+2Ø,14
Ø35
     DrawF e^(X/Ø6)/((X≥11)(X≤15))+3Ø,14
Ø36
     DrawF e^{((-X+3\emptyset)/6)}/((X \ge 16)(X \le 24))+5, 14
```

# **Spotlight Crawling on the Wall**

A rotating beacon is 10 feet from a long straight wall. The beacon makes 1 revolution every 4 seconds. The light hits the wall at a distance of d feet from the point P.

Write d as a function of  $\theta$ . Then write *d* as a function of *t*.

Transfer the program BEACON.8XP to your calculator to animate the scenario.



To stop the program, press the ON key. Then press **1:Quit**.

The program WALCRAWL.8XP shows the light crawling across the wall in a continuous fashion. As before, press the ON key to exit. Press 2nd MEM 7 2 2 to reset defaults.

Using right triangle definitions, students find the formula of d as a function of  $\theta$ : 10 0

 $\tan \theta = \frac{d}{10} \implies d = 10 \tan \theta$ 

If negative values of *d* correspond to distances to the left of *P*,

and positive values of d correspond to distances to the right of P, students can explore symmetry.

From right triangle trigonometry, we can find values of d in the table.

As the value of  $t \to \pm 1$ , we have  $\theta \to \pm \frac{\pi}{2}$  and  $d \to \pm \infty$ .

Since the beacon makes 1 revolution every 4 seconds, we have  $\theta = \frac{\pi}{2}t$ .

Students substitute  $\theta = \frac{\pi}{2}t$  into  $d = 10\tan\theta$  and so  $d = 10\tan(\frac{\pi}{2}t)$ .

Once you graph the function d, discuss with the class the connections between graph and the behavior of the beam shown in the movie.

- At t = 0, the beam is exactly at the point **P** perpendicular to the wall. The distance d from **P** is zero. The graph of the tangent function passes through the origin.
- At t = -0.5 and t = 0.5 there is an isosceles right triangle ( $\theta = \pm \frac{\pi}{4}$ ). The beam touches the wall at d = 10 ft from **P**.
- At t = -1 and t = 1 there is no triangle ( $\theta = \pm \frac{\pi}{2}$ ). The beam does not touch the wall.
- The value of d is undefined. The graph of the tangent function has vertical asymptotes at  $t = \pm 1$ . When t is near 0, the beam is near the point P, and moves much more slowly, to a crawl. The graph of the tangent function does not have a steep slope near the origin (comparatively).
- When  $t \to 1, \theta \to \frac{\pi}{2}$ , and  $d \to \infty$ . The beam moves quickly as its distance on the wall from **P** climbs. The graph of the tangent function has a steep slope near the asymptote t = 1.

**BEACON.8XP** 

P

10 ft

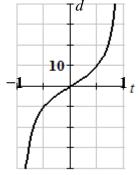
DEA	
ØØ1	Radian:Param:FnOff :GridLine :BorderColor 3
ØØ2	.Ø72→Tstep
ØØ3	-9Ø→Xmin:9Ø→Xmax:1Ø→Xscl
ØØ4	-6Ø→Ymin:6Ø→Ymax:1Ø→Yscl
ØØ5	
0006	Lbl 1
ØØ7	ClrDraw
800	<pre>For(T, -1+Tstep,1-Tstep,Tstep)</pre>
ØØ9	Line(Ø,Ø,1Øtan(πT/2),-1Ø)
Ø1Ø	End
Ø11	For(T,Ø,Ymax,1Ø)
Ø12	Line(Ø,Ø,Xmax,T)
Ø13	End
Ø14	For(T,Xmax,Xmin,-1Ø)
Ø15	Line(Ø,Ø,T,Ymax)
Ø16	End
Ø17	For $(T, Ymax, \emptyset, -5)$
Ø18	Line(Ø,Ø,Xmin,T)
Ø19	End
Ø2Ø	Goto 1
L	

d

#### WALCRAWL.8XP

- 001 Radian:Param:Fn0ff " (1 $\emptyset$ tan ( $\pi$ T/2)) (-1<T) (T≤1) {1,1,1,1,1,1,1,1,1}" -X<sub>1</sub>T " (-1 $\emptyset$ ) (-1<T) (T≤1) {1,1,1,1,1,1,1,1}" -Y<sub>1</sub>T ØØ2 ØØ3 GraphStyle(1,5) -1→Tmin:1→Tmax:.Ø72→Tstep ØØ4 ØØ5
- 006 -9Ø→Xmin:9Ø→Xmax:1Ø→Xsc1
- -6Ø→Ymin:6Ø→Ymax:1Ø→Yscl ØØ7
- 008 Lbl 1 009 DispGraph

	t	$\theta$	d	
	-1	$-\pi/2$	undefined	
	-1/2	$-\pi/4$	-10	
	0	0	0	
Ī	1/2	$\pi$ / 4	10	
	1	$\pi/2$	undefined	



Similarly, as  $t \to -1$ ,  $\theta \to -\frac{\pi}{2}$ , and  $d \to -\infty$  using symmetry.