# "How-To" <br> With Your TI-84 Plus CE Graphing Calculator Part 3 

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This handout and related accessories (programs, image files, graphical databases) can be downloaded at http://users.pfw.edu/lamaster/technology if you scroll to the presentation Create Silent Movies to Model the Real World, 29th Annual T ${ }^{3}$ International Conference, Chicago, IL, March 11, 2017. On this page you can also get links to Part 1 and 2 of this Webinar Series.

## Table of Contents

| Activity | Page |
| :--- | ---: |
| Lights! Camera! Action! - Three Ways to Produce Movies on the TI-84 Plus Family Calculator | 3 |
| Harry and Sally | 4 |
| Batman and the Joker | 5 |
| Alexander Hamilton and Aaron Burr (Student Activity) | 7 |
| Ditch Diggers | 8 |
| Bus Chase | 9 |
| Falling Objects on Different Planets | 9 |
| Falling Objects on Different Planets (Alternative) | 10 |
| Home Run | 11 |
| Roller Coaster: Animation using Function Transformations and Graph Styles | 12 |
| Math 99 Animated Movie | 13 |
| Spotlight Crawling on a Wall | 14 |

# Lights! Camera! Action! - Three Ways to Produce Movies on the TI-84 Plus Family Calculator 

## 1. Live Action

Using parametric mode, many classical story problems from traditional algebra and precalculus textbooks can be brought to life by entering the settings in four menus: Mode, Format, Y=, and Window. In the activities Harry and Sally and Batman and the Joker, the teacher directs the activity from the display calculator live before the class (with some prior setup). Students mathematize the problem to tell the teacher what value of Tmax will stop the movie to show when the main characters meet. The activity Ditch Diggers is similar. The student activity Alexander Hamilton and Aaron Burr is designed so the settings could be entered by the students to create the movie.
2. Function Transformations and Graph Styles saved with a Graphical Database (GDB) A graph database (GDB) stores the set of elements that defines a particular graph. You can recreate the graph from these elements. You can store up to 10 GDBs in variables GDB0 through GDB9 and recall them to recreate graphs.

A GDB stores five elements of a graph

- Graphing mode
- Window variables
- Format settings
- All functions in the $\mathrm{Y}=$ editor and the selection status of each
- Graph style for each $Y=$ function

GDBs do not contain drawn items (created from the Draw Menu) or stat plot definitions.
Use 2nd Draw and press the right arrow to the STO menu to enter the command StoreGDB 1 on the Home Screen to store GDB 1. To recall it, enter the command RecallGDB 1.

Some disadvantages with GDB's:

DRAW POINTS STO BACKGROUND
1:StorePic
2: RecallPic
3:StoreGDB 4: RecallGDB

- Limited to 10 GDB's. However, you can store them in a group file, and there is no limit to the number of group files you may have (up to calculator memory).
- It is easy to accidentally overwrite a GDB by storing a new one over an existing one.
- Cannot be given descriptive names so it is difficult to know what is in it.

TIP: When you store them onto a computer, you can rename the file.
You can assign a name to each GDB and then create a program to index them.

- While GDBs store most Mode and Format settings, it does not store Degree vs Radian Mode. Some examples using GDB's are Bus Chase, Falling Objects on Different Planets, Home Run? and Roller Coaster.


## 3. Use a program

There is no limit to the number of programs you may have (up to calculator memory) and these can be given somewhat descriptive names. Much more control is possible, in particular, setting mode to Degree or Radians, computer generated imagery (CGI) such as random animated fireworks and flashing lights (Math 99 Animated Movie) or replaying automatically using an infinite loop (Math 99 Animated Movie and Spotlight Crawling on a Wall).

## Harry and Sally

Adapted from an Algebra 1 Textbook: Harry the Hare and Sally the Tortoise have a race. Sally travels at a slow but steady speed of 1.6 mph . Harry travels at constant speed of 40 mph , but gives Sally a 12 hour head start. If the finish line is 21 miles away, who will win the race?

We will make a silent movie of this scenario to bring this iconic fable to life.
Let Harry travel along the line $y=4$ and Sally travel along the line $y=2$.

1. Press MODE.

Highlight parametric mode and simultaneous graphing.

2. Press WINDOW and enter these settings. Press $\mathrm{Y}=$ and enter these equations. Press GRAPH.


|  |
| :---: |



Use the 21 mile marker for the finish line.

A Look Behind the Scenes: What is happening?
Press MODE, highlight DOT-THICK.
Press GRAPH. Press TRACE and $>$.
Type 12 to jump to $\mathrm{T}=12$ hours.
Press ; to move between graphs.
In THICK (or CONNECTED) Graphing, the TI-84 connects these dots with lines.

Footsteps for each graph are connected simultaneously, so Harry will graph remarkably faster than Sally.

Lowering Tstep adds more footprints. This will slow down the action.


In Dot mode, TRACE from $\mathrm{T}=12$ to $\mathrm{T}=12.5$ with $\mathrm{TStep}=0.5$. Compare the leaps between footprints for the Hare and the Tortoise.

Set Tstep $=0.1$. The distance between footprints is smaller and more footprints appear.


In Dot mode, TRACE from $\mathrm{T}=12.4$ to $\mathrm{T}=12.5$ in steps of TStep $=0.1$.

## Class Activity: Use the movie to heighten student engagement

Set up your calculator as in Step 2 (with THICK or CONNECTED and Tstep = 0.5). Present the problem to students. Display to the class the movie on your calculator.
(Students are not using their calculator in this activity to create the movie.).

TIP: To replay the movie without disclosing the formulas in the $\mathrm{Y}=$ Editor

1. Press GRAPH
(if not already on the Graph Screen).
2. Press DRAW.
3. Press 1:ClrDraw.

NORMAL FLOAT GUTO REAL RADIAN MP
DRAW POINTS STO BACKGROUND 1:ClrDraw < One way to replay the movie. 2:Line ( 3:Horizontal 4:Vertical

- Who won? Set Tstep $=0.05$ to slow down the movie. Press GRAPH.
- Find when they meet, i.e., When Harry met Sally $)$

Replay and ask the class to tell you when to pause the action when they meet (by pressing ENTER).
(This usually is not easy without Jedi skills.)
Easier: Have the class write formulas for each racer and solve equations algebraically:

$$
\begin{aligned}
1.6 t & =40(t-12) \\
1.6 t & =40 t-480 \\
38.4 t & =480 \\
t & =\frac{480}{38.4}=12.5
\end{aligned}
$$

Assign Tmax $=12.5$, Tstep $=0.5$, and press GRAPH.

- Optional: ask the class to generate additional questions.
o At what mile marker did they meet?
Answer: 20 miles

o How long did it take each to cross the finish line?
Answer: Sally took $13.125 \mathrm{hr}=13 \mathrm{hr}, 7.5 \mathrm{~min}$;
Harry took $12.525 \mathrm{hr}=12 \mathrm{hr}, 31.5 \mathrm{~min}$
o What are the values of $t$ for which Harry is in the lead? How many seconds is this?
Answer: From $t=12.5$ to $t=12.525$.
This was the last $0.025 \mathrm{hr}=1.5 \mathrm{~min}=90$ seconds of the race!


## Batman and the Joker

This problem is similar to the Harry and Sally problem in that you can direct it from the Teacher Calculator while engaging students in the discussion.

Batman is traveling 200 mph due East when he sees the Joker coming directly toward him 27 miles away, traveling at 70 mph West. How long until they meet?

1. Set up your TI-84CE.

Highlight parametric mode and simultaneous mode.
2. Press $Y=$ and enter these equations: $\square-0 X_{1 \tau} \mathbf{E 2 0 0 T}$

3. Set the window.

```
Tmin=0
Tmax=0.2 (Set Tmax to be the length of time, in hours, you want to see them driving)
Tstep=0.005 (adjust Tstep to taste; the higher the Tstep, the faster the movie.)
Xmin=0
Xmax=27 (set Xmax to be the initial distance they are apart.)
Xscl=1
Ymin=0
Ymax=10
Yscl=1
```

Change the above movie to play for Tmax $=0.05$ hours (or consider using 60ths of an hour) Show the class that the movie stopped too soon. Bummer.
4. Now ask, "When shall I stop the clock so that the movie ends exactly when they meet?" Students set up the equation $200 t=27-70 t$ and solve for $t$ algebraically.
5. Once the solution $t=0.1$ is found, change Tmax $=0.1$. Press GRAPH to replay.


Here's another one, similar to the above:
Batman's speed is 185 mph , Joker's speed is 55 mph and they are initially 60 miles apart.

## Alexander Hamilton and Aaron Burr

Alex and Aaron meet on the dueling ground，each holding a Super Soaker water pistol．They stand back to back and simultaneously walk until they are 20 ft apart．Alex walks 1 ft per sec West，while Aaron （a bit jittery）walks 1.5 ft per sec East．How many seconds before the distance between them is 20 ft ？

First set up your TI－84CE to simulate and explore．
1．Highlight parametric mode and simultaneous mode．


2．Press $\mathrm{Y}=$ to enter equations．
－X1т日1．5T
Y1t日4
－X2t日－T
Y2t日4

3．After 10 seconds，how far apart are they？ $\qquad$ ft

| MORMAL FLOAT AUTO REAL RADIAN MP |
| :--- |
| WINDOW |
| Tmin $=0$ |
| Tmax $=10$ |
| Tstep $=1$ |
| $X \min =-20$ |
| $X \max =20$ |
| $X s c l=1$ |
| $Y$ min $=0$ |
| $Y \max =10$ |
| $Y s c l=1$ |

4．In $t$ seconds，how far apart are they？ $\qquad$ （Your answer involves t．）

Aaron Burr walked $\qquad$ ft ．（Your answer involves $t$ ．）
Alex Hamilton walked $\qquad$ ft ．（Your answer involves $t$ ．）

5．Write an equation to determine the value of $t$ which makes their distance apart equal to 20 ft ． Solve the equation．Set Tmax equal to this value of $t$ and press GRAPH．

Complete：
They are 20 ft apart in $t=$ $\qquad$ seconds； Aaron Burr walked $\qquad$ ft and Alex Hamilton walked $\qquad$ ft

## Ditch Diggers

Dan Meyer incorporates video at http://threeacts.mrmeyer to teach with three-act math tasks. (See http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/ for his approach.)

## Act One:

Introduce the story/task using as few words as possible.

In this example:
Two ditch diggers start digging towards each other from opposite directions.

The three acts for this movie are available at http://threeacts.mrmeyer.com/ditchdiggers/; however,
 this movie can be simulated on the TI-84 Family calculator.

See Dig In to the Real World with the TI-84 Plus CE at http://users.ipfw.edu/lamaster/technology/ from the 27th Annual T ${ }^{3}$ International Conference for details on how to incorporate the calculator into this activity.

Act Two: Coordinates are given for Digger $1\left(\mathrm{X}_{1 \mathrm{~T}}, \mathrm{Y}_{1 \mathrm{~T}}\right)$ and Digger $2\left(\mathrm{X}_{2 \mathrm{~T}}, \mathrm{Y}_{2 \mathrm{~T}}\right)$ for the first five days to develop the following formulas:


Act Three: Complete the task. Set up a sequel/extension. Spoiler: The diggers do not meet.


## Bus Chase

This problem was published in COMAP Consortium Number 42 in Summer 1992.
A man who is late for work is approaching a bus．When he is 25 meters from the bus it starts to pull away with an acceleration of 1.0 meter per second per second．He sees this and begins to run at 6 meters per second．Will he catch the bus？If not，how close will he come？

First we will represent the position of the man and the bus（relative to the man＇s starting point）on vertical lines on the screen．After $t$ seconds the man＇s distance from his starting point is $6 t$ and the bus＇s distance from the man＇s starting point is $25+0.5 t^{2}$ ．

1．Use $X_{1 T}$ and $Y_{1 T}$ to represent the man＇s motion and $X_{2 T}$ and $Y_{2 T}$ to represent the bus＇s motion． Make the man and the bus move on the vertical line $x=-3$ ．Set the equations and the window to do this．Use SIMUL mode．Use O Line Style for the bus and dotted line style for the man．
2．Trace to see if he catches the bus．
3．Graph the distance between the bus and the man versus time $t$ ．To do this let $X_{3 \mathrm{~T}}=\mathrm{T}$ and $\mathrm{Y}_{3 \mathrm{~T}}=\mathrm{Y}_{1 \mathrm{~T}}-\mathrm{Y}_{2 \text { т }}$ ．Keep the window set so that you can see the vertical motions and the plot of $\mathrm{Y}_{3 \mathrm{~T}}$ ．

Screenshots from the Bus Chase problem：

| mormal float auto real radian mp | $\square$ |
| :---: | :---: |
| ```Plot1 Plot2 Plot3 -0X1т日-3 Y14日6T 10X2т日-3``` |  |
|  |  |






## Falling Objects on Different Planets

Teacher：＂Three balls will be dropped from a height of 100 meters on each of the planets，Mars，Earth，and Jupiter．＂
Show the TI－84 movie of the balls dropping．Do not show formulas yet．
＂Discuss in your groups：
What is similar about each？What is different？
Which planet goes with which graph？＂
Prior to class set up the calculator using the screens to the right．
A GDB is an excellent way to have this ready to go for class．
MORMAL FLOAT AUTO REAL RADIAN MP－

| Plot1 Plot2 Plot3 $\mathrm{X}_{1} \mathrm{Bl}_{1}$ |
| :---: |
| $\mathrm{Y}_{1 \mathrm{~T}} \mathrm{E}-1.86 \mathrm{~T}^{2}+100$ |
|  |
| $\begin{array}{rl} Y_{3 T} & 日-11.44 T^{2}+100 \\ X_{4 \mathrm{~T}} & = \\ Y_{4 \mathrm{~T}} & = \end{array}$ |

Students may bring up the following similarities：
－Each has the same initial height．
－As they fall，they move faster and faster．
Students may bring up the following differences：
－Length of time to drop．
－Rate of acceleration．

## Match：

The slowest is Jupiter，fastest is Mars，and Earth is in between．
The larger the planet＇s mass，the larger the acceleration due to gravity．

NORMaL FLOAT RUTO REAL RADIAN MP
WINDOW
Tmin＝0
Tmax $=8$
Tstep＝0．05
Xmin＝－2
$X_{m a x}=8$
Xscl＝1
Ymin＝0
$Y_{\max }=150$
$\mathrm{Yscl}=100$

Optional（based on the level of your students）：Help students construct the formulas for each．

## Falling Objects on Different Planets (Alternative)

Based on students' level, interest, or available class time, the previous activity can be extended. Instead of setting up the calculator with only the three balls dropping, use the screens to the right, where the last three graphs are deselected (not highlighted) and use the more expanded viewing window $-8 \leq x \leq 8$.

As before, play the movie while asking students to find similarities, differences, and to match the planet with the graph.

Enter the command Fn0n $4,5,6$ to turn on the last three functions without disclosing their formulas in the $\mathrm{Y}=$ Editor.
(Use VARS Y-VARS 4:On/Off to paste the FnOn command on the home screen.) Press DRAW 1:ClrDraw to refresh the movie and press GRAPH.

Production Tip: If you select ClrDraw immediately from leaving the Graph Screen it will immediately redraw. If you select it from any other menu, the command is pasted on the Home Screen and you must then press GRAPH to refresh the screen.

Provide these three formulas: $a(t)=-1.86 t^{2}+100$,

$$
\begin{aligned}
& b(t)=-4.9 t^{2}+100 \\
& c(t)=-11.44 t^{2}+100
\end{aligned}
$$

Now ask students to match the formula to the planet and explain their reasoning. Students may connect the planet with the time it takes for the ball to hit the ground (which now you can ask them to find, if class time permits). Other students may point out the leading coefficient has vertically stretched or compressed the graph and affects the rate at which the ball is falling.

Yet even another approach: Instead of providing the formulas, ask students to construct a formula for $h(t)$, the height of the ball above ground in $t$ seconds for each planet if $h(t)=\frac{1}{2} g t^{2}+h_{0}$, where $g$ is the acceleration due to gravity and $h_{0}$ is the initial height. For Mars, Earth, and Jupiter, $g=-3.72,-9.8$, and -22.88 meters per $\mathrm{sec}^{2}$, respectively.

## Home Run？

Adapted from a problem by Mike Koehler published in the Fall 1993 issue of Eightysomething！
The＂Green Monster，＂the left field wall in Fenway Park in Boston is 37 feet high．Its distance from home plate is 379 feet．A baseball is hit by a Boston Red Sox player when it is 3 feet from the ground with an initial velocity of 123 feet per sec at an angle of elevation of $32^{\circ}$ ．

A wind of $3 \mathrm{mph}(4.4 \mathrm{ft}$ per sec）is blowing towards the batter．The ball is on a path that will take it to the deepest part of the Green Monster， 379 feet away．Is it a home run？

The initial force is resolved into horizontal（ $x$ ）and vertical（ $y$ ） components using trigonometry．

$$
\begin{aligned}
& V_{x}=123 \cos 32^{\circ} \\
& V_{y}=123 \sin 32^{\circ}
\end{aligned}
$$



The horizontal parametric equation is $X(t)=V_{x} T+W$ ，where $V_{x}$ is the horizontal component of the initial velocity，$T$ is the time，and $W$ is the other horizontal forces working，such as the wind．
The vertical parametric equation is $Y(t)=-16 T^{2}+V_{y} T+S_{0}$ ，where $V_{y}$ is the vertical component of the initial velocity，$T$ is the time，and $S_{0}$ is the starting height．
Enter the equations：
The ball：－0X1т日123cos $\left(32^{\circ}\right) \mathrm{T}-4.4 \mathrm{~T}$
$Y_{1 \text { т }} \mathrm{E}\left(-16 \mathrm{~T}^{2}+\left(123 \sin \left(32^{\circ}\right) \mathrm{T}+3\right) /(\mathrm{T}<3.85)\right.$


The wall： $\mathbf{~ - X 2 т 日 3 7 9 ~}$
The quantity（ $\mathrm{T}<3.85$ ）is 1 if true， 0 if false．
$\mathrm{Y}_{2 \mathrm{t}} \mathrm{\theta} 37 \mathrm{~T}(\mathrm{~T} \leq 1) \quad$ A division by 0 occurs for $t \geq 3.85$ ，so the domain is restricted to $t<3.85$ ．


The quantity（ $\mathrm{T} \leq 1$ ）is 1 if true， 0 if false．
The wall will rise from the ground until，when $t=1$ ，its height is 37 ft ．and thereafter rise no more． For $t>1$ the height of the wall is 0 ．
The multiplication of $(\mathrm{T} \leq 1)$ creates a line connecting $(379,37)$ to $(379,0)$ when $t=1$ ．
Press WINDOW．Use the settings shown． Press GRAPH．

| MORMAL FLOAT AUTO REAL DEGREE CL | ［］ |
| :---: | :---: |
| WINDOW |  |
| Tmin $=0$ |  |
| Tmax＝5 |  |
| Tstep＝0．05 |  |
| Xmin＝0 |  |
| $X_{\text {max }}=400$ |  |
| Xscl＝100 |  |
| Ymin＝0 |  |
| $Y_{\text {max }}=100$ |  |
| $\mathrm{Yscl}=10$ |  |



The graph of $\mathrm{Y}_{\text {зт }}$ has a restricted domain of $t>3.8$ ．After playing the movie，you can see what it does！

```
|-0X (т目(758-X1т)/(T\geq3.80)
\(Y_{3 \text { т }} \mathrm{E}\left(-16 \mathrm{~T}^{2}+\left(123 \sin \left(32^{\circ}\right) \mathrm{T}+3\right) /(\mathrm{T} \geq 3.80)\right.\)
```

The graph of $\mathrm{Y}_{4 \text { т }}$ is just for fun． Position your cursor over the＝sign， press ENTER to highlight it， and press GRAPH．

Store this into a GDB to recall later．

## Roller Coaster

Animations can be done using Graph Styles and transformations of functions．The GraphStyle in Y6 creates a ball that has been raised to sit just above the＂rollercoaster＂sinusoidal path．The ball floats above the roller coaster track because the sine curve in Y3 has been translated vertically．

Note also that the sun and the bird are done with lists and restricted domains so that multiple transformations can be done as one graph．This gives the student the ability to graph more than 10 functions at a time．

|  |  |
| :---: | :---: |
| ```Plot1 Plot2 Plot3 납⽇( \(\{4,2,1, .4\}-(X+10)^{\wedge} 2\) )+5 VY2日- \(\sqrt{\left(\{4,2,1, .4\}-(X+10)^{\wedge}\right.}\) 2) +5 - \(\mathrm{H}_{3}\) 日 \(2 \sin (.5 X)-3\) - \Y4日 \(\left(-8(X-8)^{\wedge} 2+5\right) /((X \leq 8.4\) and \(X \geq 7.6\) ) - Y \(_{5 \text { 日 }}\left(-8(X-8.8)^{\wedge} 2+5\right) /((X \geq 8\) .4 and \(X \leq 9.2\) ))``` |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


| XORMAL FLOAT AUTO REGL RADIAN CL |
| :--- | :--- |
| WINDOW |
| Xmin $=-13.2$ |
| Xmax $=13.2$ |
| Xscl $=1$ |
| Ymin $=-8$ |
| Ymax $=8$ |
| Yscl $=1$ |
| Xres $=1$ |
| $\Delta X=0.1$ |
| TraceStep $=0.2$ |




## Math 99 Animated Movie

The program shown below was created for an Advanced Calculator Topics course (numbered MATH 99). It contains animated features such as fireworks and flashing lights to highlight the kind of CGI movies possible using a program.

These fireworks randomly appear on the screen:


After the title MATH 99 unveils, the animation automatically repeats indefinitely. Press the ON key to interrupt the program. Then press 1:Quit.

After running a program, you may wish to Reset Defaults.


Press 2nd MEM 722.

Below is the code for the program MATH99.
It can be downloaded on the Website listed on the cover of this handout.

| øø1 | Radian |
| :---: | :---: |
| øめ2 | Fn0ff :Plots0ff :Axes0ff |
| Ø03 | ClrDraw |
| Ø04 | Background0n 22 |
| ø05 | $\emptyset \rightarrow X \mathrm{~min}$ |
| 006 | $94 \rightarrow$ Xmax |
| Ø07 | $\emptyset \rightarrow X s c 1$ |
| Ø08 | $\emptyset \rightarrow$ Ymin |
| Ø09 | $62 \rightarrow Y$ max |
| ø10 | $\emptyset \rightarrow Y \mathrm{Sc} 1$ |
| 011 | Lb 1 |
| Ø12 |  |
| Ø13 | For (P, 1, 5, 1) |
| Ø14 | C1rDraw |
| Ø15 | Text (2,10, "COLLEGE OF THE REDWOODS") |
| 016 | Text (20,10, "PRESENTS . . . . . ") |
| 017 | randInt $(35,6 \varnothing) \rightarrow J$ |
| 018 | randInt (10,30) $\rightarrow$ K |
| 019 |  |
| Ø2ø | For ( $0,1,9,1$ ) |
| Ø21 | randint (10,21) $\rightarrow \mathrm{L}$, |
| Ø22 | Line (J, K, randInt (30,65), randInt (5,45), 1, L) |
| Ø23 | End |
| Ø24 |  |
| 025 | End |
| ø26 | C1rDraw |
| ø27 | Text (2,1ø,"COLLEGE OF THE REDWOODS") |
| 028 | Text (2ø,1ø, "PRESENTS...........") |
| 029 | Line (15, $\varnothing$, 15, 43, 1, 16, 2) |
| ø3ø | Line (15.5, $0,15.5,39,1,16,2)$ |
| Ø31 | Line (16, $\varnothing, 16,33,1,16,2)$ |
| Ø32 | DrawF $e^{\wedge}(\mathrm{X} / \varnothing 6) /((\mathrm{X} \geq \varnothing 8)(\mathrm{X} \leq 15)), 14$ |
| Ø33 | DrawF $e^{\wedge}(\mathrm{X} / \varnothing 6) /((\mathrm{X} \geq$ ¢9) $(\mathrm{X} \leq 15))+1 \varnothing, 14$ |
| Ø34 | DrawF $e^{\wedge}(\mathrm{X} / \varnothing 6) /((\mathrm{X} \geq 1 \varnothing)(\mathrm{X} \leq 15))+2 \varnothing, 14$ |
| Ø35 | DrawF $e^{\wedge}(\mathrm{X} / \varnothing 6) /((\mathrm{X} \geq 11)(\mathrm{X} \leq 15))+3 \varnothing, 14$ |
| ø36 | DrawF $e^{\wedge}((-X+3 \varnothing) / 6) /((X \geq 16)(X \leq 24))+5,14$ |
| Ø37 | DrawF $e^{\wedge}((-X+3 \varnothing) / 6) /((X \geq 16)(X \leq 22))+16,14$ |
| Ø38 | DrawF $e^{\wedge}((-X+3 \varnothing) / 6) /((X \geq 15)(X \leq 21))+26,14$ |

```
039 Line(30,39,27,22)
\varnothing4\varnothing DrawF (.25(X-36)^2+31)/((X\geq30) (X\leq42))
Ø41 Line(42,39,39,22)
042 Line (49,39, 43, 22)
043 Line (49,39,53, 22)
044 Line (46, 28,51, 28)
045 Line (58,39,69,39)
\varnothing46 Line (63,39,60, 22)
047 Line(73,39,70,22)
Ø48 Line (82,39,79, 22)
049 Line (72,31, 81,31)
\emptyset\emptyset For(I, 1, \varnothing2,1)
\varnothing51 StorePic 7
\emptyset52 ClrDraw
Ø53 seq (X,X,1,\varnothing\varnothing1, 1) ->L 5
054 RecallPic 7
\emptyset55 seq (X,X, 1, Øø1, 1) )L 5
\emptyset56 End
057 Circle(46, 14,6,18)
058 Circle(64,14,6,18)
059 Line (52, 14, 49, , 1, 18)
ø6ø Line(7\varnothing,14,67,\varnothing,1,18)
\varnothing61 seq (X,X,1,475,1)->L5
Ø62 ClrDraw
\varnothing63 Text(1\varnothing,\varnothing8,"ADV. CALCULATOR TOPICS")
\varnothing64 Text(5\emptyset,2\varnothing,"WEDNESDAYS 1-3 PM"
\emptyset65 Text (3\emptyset,12,"STUART MOSKOWITZ")
Ø66 DrawF (Ø8sin(.2X)+15)/((X\geq1\varnothing) (X\leq84))
Ø67 seq(X,X,1,øø2,1)
068 Goto 1
```


# Spotlight Crawling on the Wall 

A rotating beacon is 10 feet from a long straight wall.
The beacon makes 1 revolution every 4 seconds.
The light hits the wall at a distance of $d$ feet from the point $P$.
Write $d$ as a function of $\theta$.
Then write $d$ as a function of $t$.


Transfer the program BEACON.8XP to your calculator to animate the scenario.


The program WALCRAWL.8XP shows the light crawling across the wall in a continuous fashion. As before, press the ON key to exit. Press 2nd MEM 722 to reset defaults.

Using right triangle definitions, students find the formula of $d$ as a function of $\theta$ :
$\tan \theta=\frac{d}{10} \Rightarrow d=10 \tan \theta$


## BEACON.8XP

```
ø01 Radian:Param:Fn0ff :GridLine :BorderColor 3
    .072->Tstep
    -9\emptyset->Xmin: 9 ¢ Xmax: 1 }|\mathrm{ Xsc 1
    -6\varnothing->Ymin: 6\varnothing->Ymax:1 }\varnothing->Y\mathrm{ Yc 1
    Lb1 }
    ClrDraw
    For(T, -1+Tstep,1-Tstep,Tstep)
    Line ( }\varnothing,\varnothing,1\emptyset\operatorname{tan}(\pi\textrm{T}/2),-1\varnothing
    End
    For(T,\varnothing,Ymax, 1\varnothing)
    Line (\varnothing, },\mathrm{ Xmax,T)
    End
    For (T,Xmax,Xmin, -1\varnothing)
    Line ( }\varnothing,\varnothing,T,Ymax
    End
    For (T,Ymax, \varnothing, -5)
    Line( ( , }\varnothing,Xmin,T
    End
    Goto }
øø3 -9 \(9 \rightarrow\) Xmin: \(90 \rightarrow X m a x: 1 \varnothing \rightarrow X s c 1\)
øø4 -6ø - Ymin: \(6 \varnothing \rightarrow\) Ymax: \(1 \varnothing \rightarrow\) Ysc 1
06 Lb1 1
807 C1rDraw
For(T,-1+Tstep,1-Tstep,Tstep)
\(\varnothing \varnothing 9\) Line ( \(\varnothing, \varnothing, 1 \varnothing \tan (\pi \mathrm{~T} / 2),-1 \varnothing)\)
olo End
Line \(\left(\varnothing, \varnothing\right.\), Xmax \(^{1}\),
\(\varnothing 13\) End
ه14 For(T,Xmax,Xmin,-1ø)
\(\varnothing 15\) Line ( \(\varnothing, \varnothing, \mathrm{T}, \mathrm{Ymax})\)
\(\varnothing 16\) End
Ø17 For (T,Ymax, Ø, -5)
Ø18 Line (Ø, Ø, Xmin, T)
ø19 End
20 Goto 1
```


## WALCRAWL.8XP

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øø1 Radian:Param:Fn0ff

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øø1 Radian:Param:Fn0ff

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\varnothing\varnothing3 "(-1\varnothing)(-1<T)(T\leq1){1,1,1,1,1,1,1,1,1,1}"->Y', '

```
```

\varnothing\varnothing3 "(-1\varnothing)(-1<T)(T\leq1){1,1,1,1,1,1,1,1,1,1}"->Y', '
Ø04 GraphStyle(1,5)
Ø04 GraphStyle(1,5)
\varnothing\varnothing5 -1->Tmin:1->Tmax:.ø72->Tstep
\varnothing\varnothing5 -1->Tmin:1->Tmax:.ø72->Tstep
ø\varnothing6 -9\emptyset->Xmin:9\emptyset->Xmax:1\varnothing->Xsc1
ø\varnothing6 -9\emptyset->Xmin:9\emptyset->Xmax:1\varnothing->Xsc1
\varnothing\varnothing7 -6\varnothing->Ymin:6\varnothing->Ymax:10->Ysc1
\varnothing\varnothing7 -6\varnothing->Ymin:6\varnothing->Ymax:10->Ysc1
øø8 Lb1 1
øø8 Lb1 1
\varnothingø9 DispGraph

```
```

\varnothingø9 DispGraph

```
```

To stop the program, press the ON key. Then press 1:Quit.

If negative values of $d$ correspond to distances to the left of $P$, and positive values of $d$ correspond to distances to the right of $P$, students can explore symmetry. From right triangle trigonometry, we can find values of $d$ in the table.
As the value of $t \rightarrow \pm 1$, we have $\theta \rightarrow \pm \frac{\pi}{2}$ and $d \rightarrow \pm \infty$.
Since the beacon makes 1 revolution every 4 seconds, we have $\theta=\frac{\pi}{2} t$.
Students substitute $\theta=\frac{\pi}{2} t$ into $d=10 \tan \theta$ and so $d=10 \tan \left(\frac{\pi}{2} t\right)$.
Once you graph the function $d$, discuss with the class the connections

| $t$ | $\theta$ | $d$ |
| :---: | :---: | :---: |
| -1 | $-\pi / 2$ | undefined |
| $-1 / 2$ | $-\pi / 4$ | -10 |
| 0 | 0 | 0 |
| $1 / 2$ | $\pi / 4$ | 10 |
| 1 | $\pi / 2$ | undefined | between graph and the behavior of the beam shown in the movie.

- At $t=0$, the beam is exactly at the point $\boldsymbol{P}$ perpendicular to the wall. The distance $d$ from $\boldsymbol{P}$ is zero. The graph of the tangent function passes through the origin.
- At $t=-0.5$ and $t=0.5$ there is an isosceles right triangle $\left(\theta= \pm \frac{\pi}{4}\right)$.

The beam touches the wall at $d=10 \mathrm{ft}$ from $\boldsymbol{P}$.

- At $t=-1$ and $t=1$ there is no triangle $\left(\theta= \pm \frac{\pi}{2}\right)$. The beam does not touch the wall.

The value of $d$ is undefined. The graph of the tangent function has vertical asymptotes at $t= \pm 1$.

- When $t$ is near 0 , the beam is near the point $\boldsymbol{P}$, and moves much more slowly, to a crawl. The graph of the tangent function does not have a steep slope near the origin (comparatively).
- When $t \rightarrow 1, \theta \rightarrow \frac{\pi}{2}$, and $d \rightarrow \infty$. The beam moves quickly as its distance on the wall from $\boldsymbol{P}$ climbs. The graph of the tangent function has a steep slope near the asymptote $t=1$.

- Similarly, as $t \rightarrow-1, \theta \rightarrow-\frac{\pi}{2}$, and $d \rightarrow-\infty$ using symmetry.

