Bright Colors and More: See What the TI-84 Plus C Silver Edition Graphing Calculator Can Do

25th Annual T^3 International Conference
Philadelphia, PA

Saturday, March 9, 2013
1:00 p.m. - 2:00 p.m.
2:15 p.m. - 3:45 p.m.
Marriott Franklin 2

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Investigation 1: Fun with Simplifying Rational Expressions

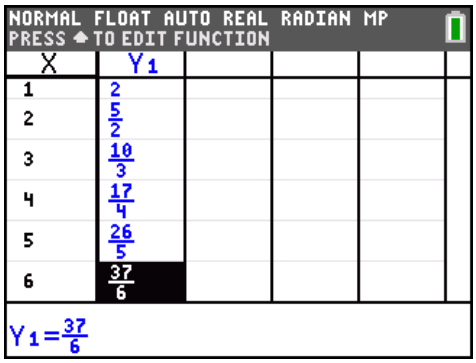
This activity provides opportunities to foster these *Common Core State Standards for Mathematical Practice*:
MP#7 Look for and make use of structure.
MP#8 Look for and express regularity in repeated reasoning.

1. Enter $x+\frac{1}{x}$ in Y1. For the shortcut FRAC menu, press ALPHA [F1].

   Set the table to start at 1, climb in steps of 1, automatically display the input and output.
   Press 2nd [TABLE].

   Using only the table of values, discuss the following

   - What do you expect the next value to be?
   - What pattern(s) do you see with the numerators? List as many patterns as you can find.
   - Use the arrow key to continue the table to see if your prediction is correct.



   New with TI-84 Plus C:
   - Status Bar has context help.
   - Color coordinated with its heading (top row) and its function expression (bottom row).
   - MathPrint™ mode enhancements:
     - Stacked fractions will now display instead of thick-bar fractions in MathPrint™ mode.
     - MathPrint™ templates are now available for expressions edited from the bottom line.
   - More functions in view!
   - More lines of data! (Using stacked fractions in MathPrint™ mode reduces the number of rows.)
   - Separator lines enhance readability

2. Use algebra to simplify the expression in Y1. What information does this simplified expression provide to help confirm or extend your observations in the previous question?

3. For your own enjoyment, the following delights can be explored in a similar way.

   a. Once you have a table of values for $x+\frac{1}{x}$, try $x-\frac{1}{x}$. Compare them in the same table.

   b. Explore the patterns in a table for $2+\frac{1}{x}$ (again, without simplifying). Compare with $2-\frac{1}{x}$

   c. Compare $1-\frac{1}{x+1}$ and $1+\frac{1}{x+1}$. How about $x-\frac{1}{x^2}$ and $x+\frac{1}{x^2}$?

   d. Use the summation command (press ALPHA [F2]) to enter the following in the Y= editor and compare tables, starting at $x = 0$, climbing in steps of 1:

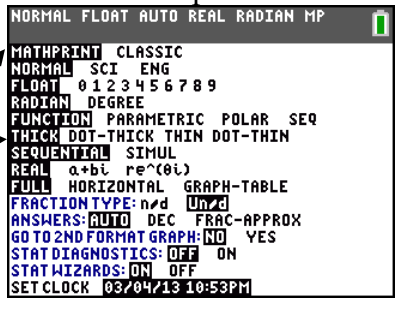
   $\sum_{I=0}^{X} 2I+1$ vs. $\sum_{I=0}^{X} 2I$ *(the rectangular numbers)* vs. $\sum_{I=0}^{X} I$ *(the triangular numbers)*

4. With $x+\frac{1}{x}$ in Y1, press MODE to change FRACTION TYPE to mixed **Un/d**. Explore the table.

New with TI-84 Plus C: Easy switch between MATHPRINT and CLASSIC.

Due to higher resolution we get more graph styles. (This setting affects all functions in Y= Editor)

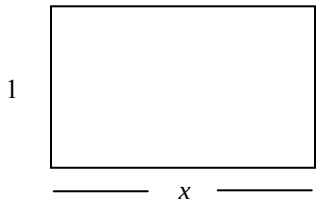
Change in color. Why?



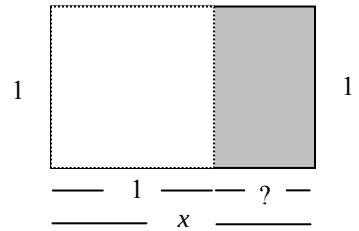
Investigation 2: Mining for Gold

In this investigation we will explore rectangles with a special property and then construct them with the TI-84 Plus C. This activity assumes students have seen similar rectangles and can use the quadratic formula to solve equations involving ratios and proportions. Students learn how to use a square to construct a golden rectangle from another given golden rectangle, using many Common Core State Standards for Mathematical Practice along the way.

1. If you are given a rectangle in which the ratio of the shorter side to the longer side is 1: x,

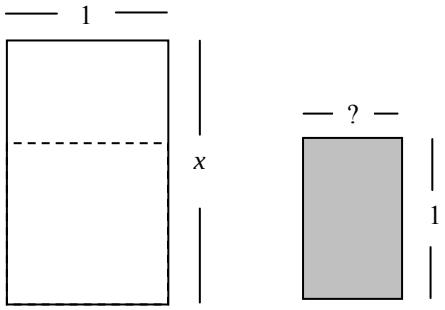


and then you divide the rectangle into a square and a new rectangle (shown shaded),



then what will be the length of the shorter side of the shaded rectangle, in terms of x?

2. Wouldn't it be awesome if the dimensions of the *original rectangle* were such that, when a square is removed, the *shaded rectangle* that remains would be the same shape, i.e., the ratios of the side lengths are the same!



Such similar rectangles are called **golden rectangles**.

a. Use the previous question to complete the boxes.
If the ratio side lengths of the two rectangles above are equal, then we have

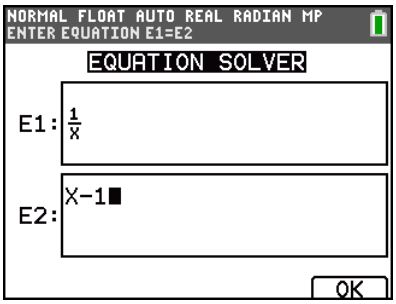
$$\frac{1}{x} = \frac{\square}{\square}.$$

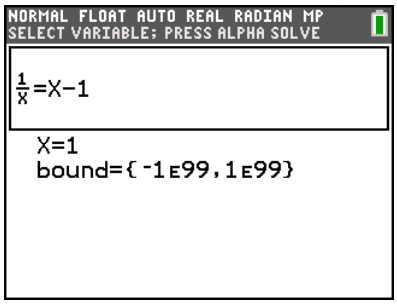
The **golden ratio** is the number ϕ that is the positive solution to that equation.

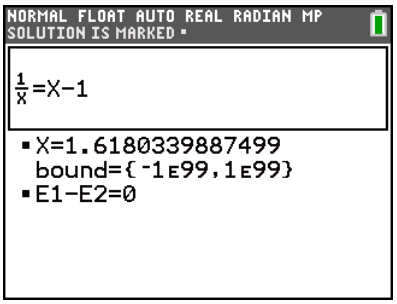
b. In the above equation, cross multiply to create an equivalent quadratic equation.
Report your quadratic equation: _____

c. Find its exact value by solving this equation algebraically for x. Spoiler alert if you flip the page!

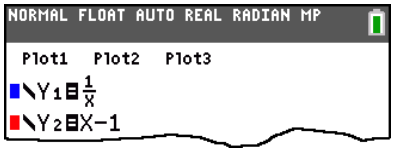
3. Confirm with the Equation Solver. Press MATH, use the UP arrow to scroll to the last item on the menu and select **B:Solver…** , and follow the directions provided in the Status Bar to approximate ϕ.



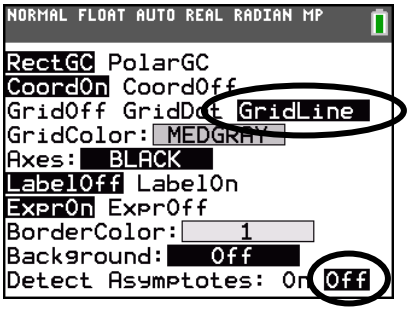




4. Solve the equation $\frac{1}{x} = \frac{x-1}{1}$ graphically for ϕ.

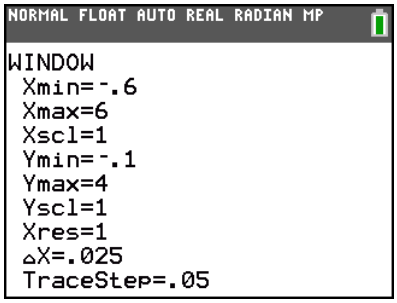


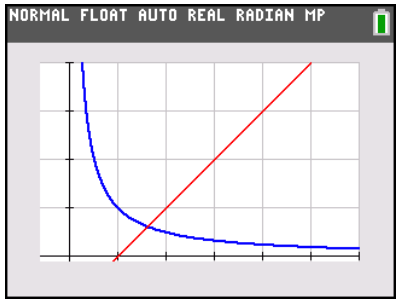
Enter the left and right side of the equation $\frac{1}{x} = \frac{x-1}{1}$ in the Y= menu.



Press 2nd [FORMAT] to display the **GridLine**.
To speed up graphing, set Detect Asymptotes to **Off**.

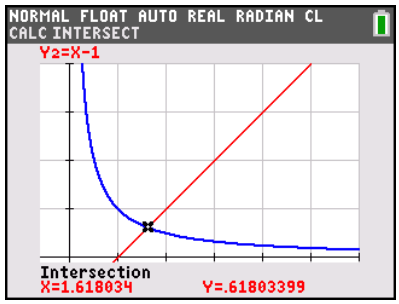
One good possibility for a viewing window is shown.



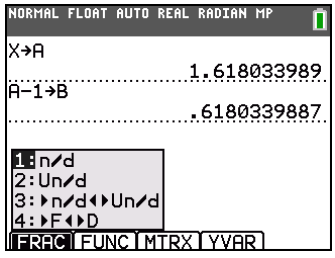


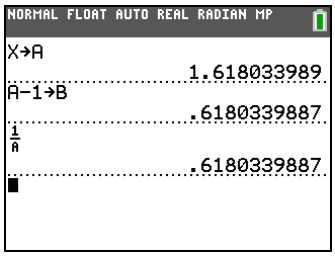
It is both "square" (providing a true geometric perspective) and "friendly" (enabling the free-moving cursor to display integer screen coordinates.)

Press 2nd [CALC], select **5:intersect**, and follow the prompts.



Press 2nd [QUIT] to get to the Home Screen.
Store the value of ϕ into the variable **A**.
Press X,T,Θ,n STO▶ ALPHA **A**
Similarly, store the *y*-coordinate, $\phi - 1$, into **B**.

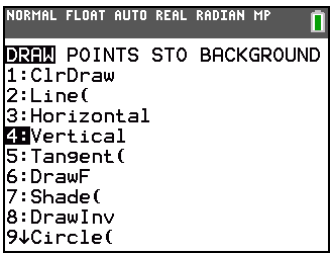


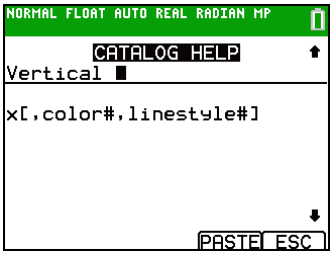


Note that $\phi - 1 = \frac{1}{\phi}$.

5. We will draw the vertical line $x = \phi$.

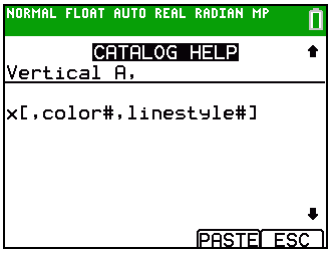
The DRAW menu can be used *interactively* for most commands when you access the menu directly from a graph. Otherwise, pressing [2nd] [DRAW] will paste the command onto the Home Screen (or, if editing a program, in the program editor).



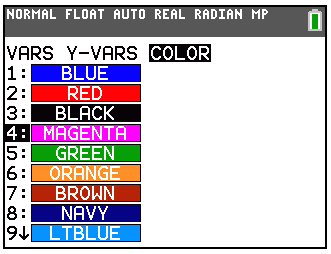


From the Home Screen, press [2nd] [DRAW].
Use [▼] to highlight **4:Vertical**.
Optionally, we can use built-in Catalog Help by pressing [+]. The Status Bar changes to green.

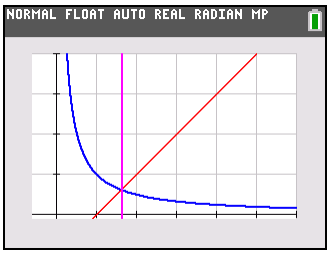
Type [ALPHA] [A] (where you have stored the value of ϕ).



Use the VARS COLOR menu to select the color MAGENTA.



Press [ALPHA] [F4] to PASTE the command on the Home Screen. Press [ENTER].



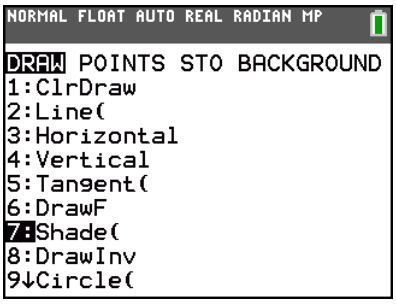
Arguments for colors can also be a color # (10-24), i.e., `Vertical A,13`.

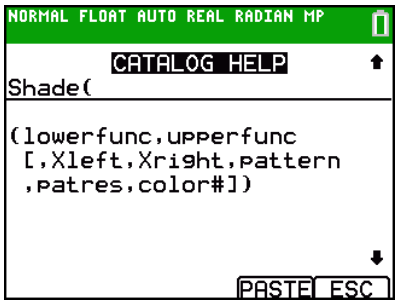
Arguments for **lineStyle#** must be a number.
For lines, there are four options:
1 - Thin ( )
2 - Thick ( )
3 - Above ( ), and
4 - Below ( ).

These are the same options available on the spinner which is displayed after pressing the STYLE soft key when you call the command interactively from the GRAPH screen.

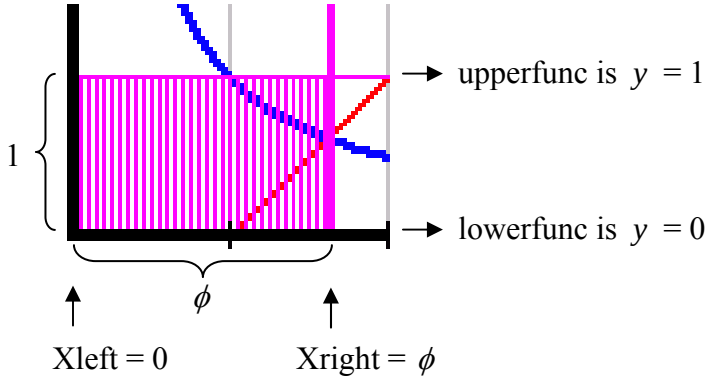
6. With the Shade command and the graphs of $y = \frac{1}{x}$ and $y = x - 1$ we can draw a golden rectangle with width 1 and length ϕ on the Graph Screen.

Press [2nd] [DRAW],
use [▼] to highlight **7:Shade(** ,
and press [+] for Catalog Help.





The arguments *lowerfunc*, *upperfunc*, *Xleft*, and *Xright* are shown below.



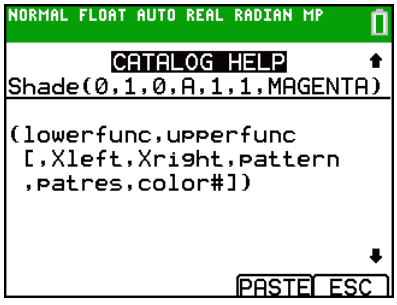
Pattern specifies one of four shading patterns.

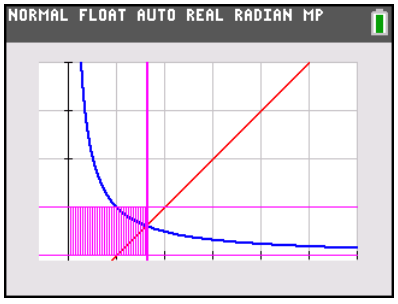
pattern=**1**	vertical (default)
pattern=**2**	horizontal
pattern=**3**	negative—slope 45°
pattern=**4**	positive—slope 45°

Patres specifies one of eight shading resolutions. →

patres=**1** shades every pixel (default)
patres=**2** shades every second pixel
patres=**3** shades every third pixel
patres=**4** shades every fourth pixel
patres=**5** shades every fifth pixel
patres=**6** shades every sixth pixel
patres=**7** shades every seventh pixel
patres=**8** shades every eighth pixel

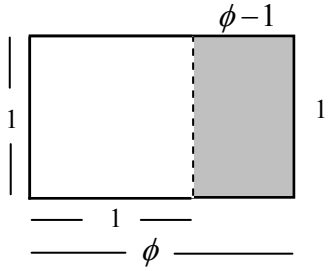
Press [ALPHA] [F4] to PASTE the command on the Home Screen. Press [ENTER].



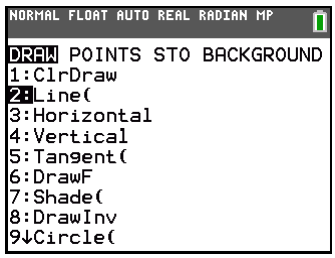


Congratulations on constructing a bona fide golden rectangle!

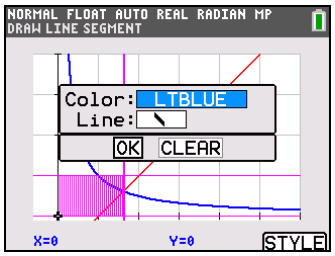
Given one golden rectangle, more can follow,
with the help of a square.



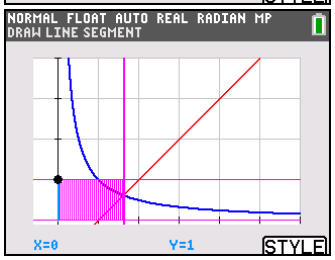
7. Draw a unit square interactively on the Graph Screen.
Press [GRAPH] [2nd] [DRAW] and use [▼] to select **2:Line** and press [ENTER].
You will be taken to the Graph Screen.

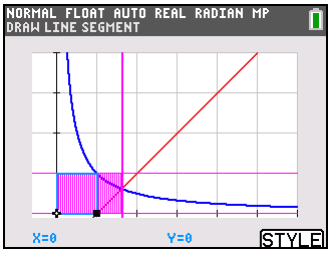


Press [ALPHA] [F5] to open the **STYLE** spinner,
use the arrow keys to select your color and line style.
Highlight OK and press [ENTER] when done.

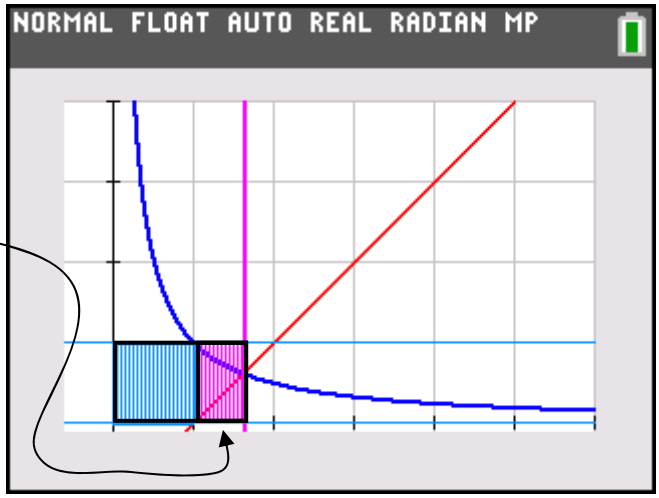


Position to (0, 0). Press [ENTER].
Use [▲] to move the cursor to (0,1). Press [ENTER].
At (0, 1), press [ENTER] to begin drawing a new segment.
Use [▶] to move the cursor to (1, 1). Press [ENTER].
Repeat until you have all four sides of the square.





Shade the contents of the unit square using
`Shade(0,1,0,1,1,1,LTBLUE)`
The rectangle next to this square is golden.

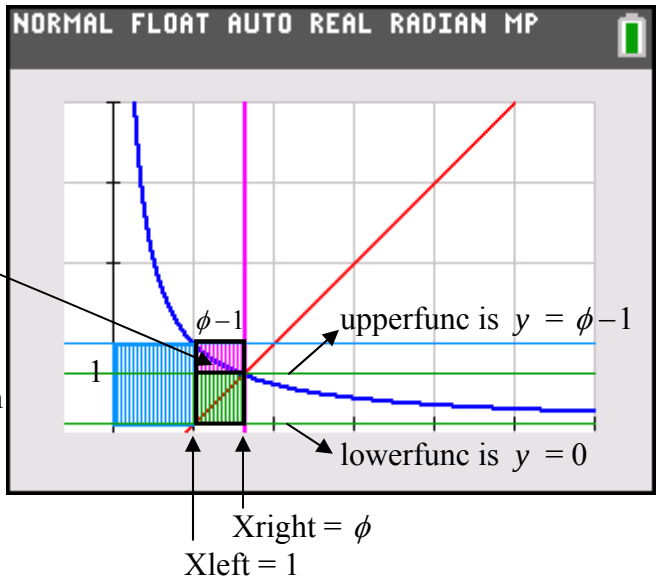


8. The procedure can be repeated to make even more golden rectangles:

Since **B** is the value of $\phi - 1$ we shade a green square using the command
`Shade(0,B,1,A,1,1,GREEN)`
The rectangle next to this (green) square will have length $\phi - 1$.

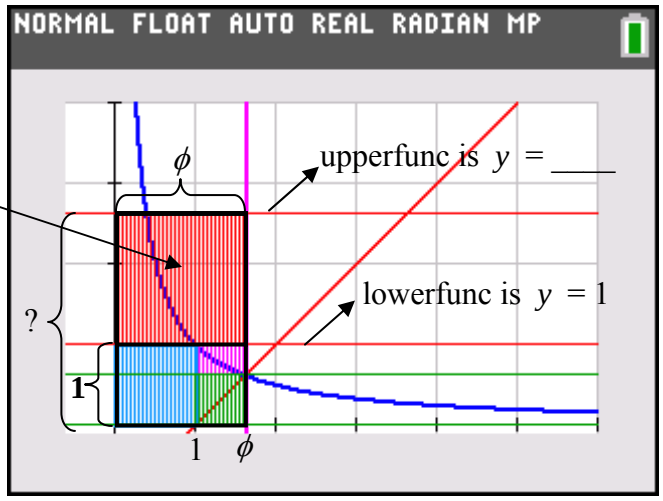
a. Show that its width is $2 - \phi$.

b. Show that the ratio $\dfrac{\text{width}}{\text{length}} = \dfrac{2-\phi}{\phi-1} = \dfrac{1}{\phi}$ results in an equation equivalent to that which you found in Step 2b at the beginning of this handout.



Extensions

1. If you already have a golden rectangle, you can also append a square to it to yield another golden rectangle.
   Use the Shade command to draw the red square.

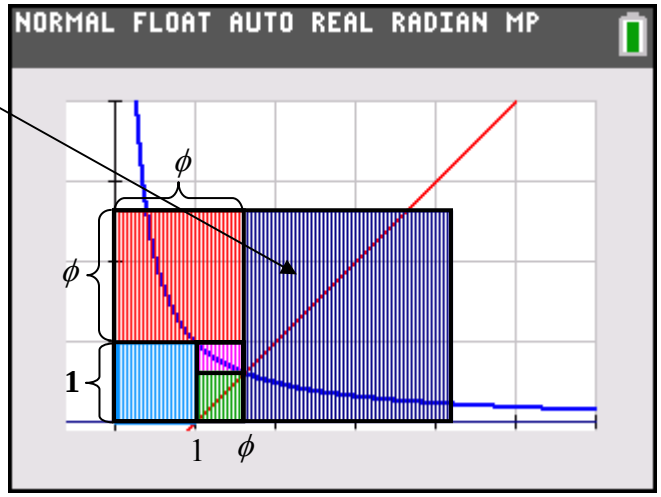


   a. You have now made a golden rectangle with width ϕ. What is its **length**?

   b. Complete the boxes: $\dfrac{\text{width}}{\text{length}} = \dfrac{\phi}{\square} = \dfrac{1}{\phi}$

   c. Cross multiply. Create a quadratic equation to show algebraically this rectangle is similar to a 1 by ϕ golden rectangle.

2. Use the Shade command to draw the navy square.



   a. You have now made a golden rectangle with width $\phi+1$. What is its **length**?

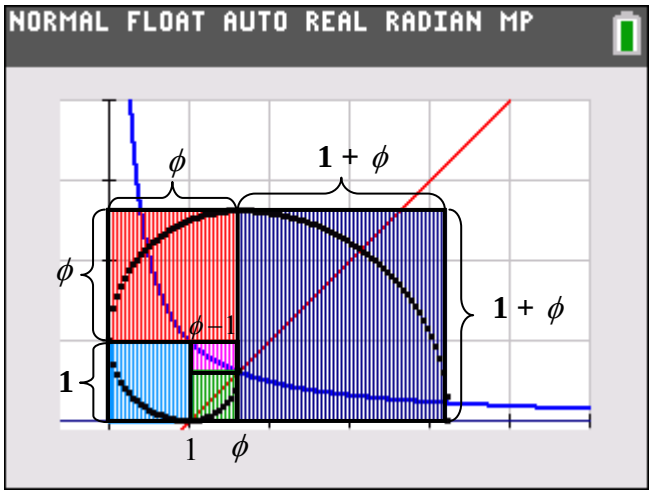
   b. Complete the boxes: $\dfrac{\text{width}}{\text{length}} = \dfrac{\phi+1}{\square} = \dfrac{1}{\phi}$

   c. Cross multiply. Create a quadratic equation to show algebraically this rectangle is similar to a 1 by ϕ golden rectangle.

3. One way the golden ratio ϕ has been defined is as follows:
   "ϕ is the positive number with this special property: to square it, you just add 1."
   Is this true? Explain.

Mining for Gold - Teacher's Notes

1. In the **Extension**, students append squares to existing golden rectangles, create the "whirling rectangles" shown to the right. They set up proportions to show $\frac{\text{width}}{\text{length}} = \frac{\phi}{\phi+1} = \frac{1}{\phi}$ and $\frac{\text{width}}{\text{length}} = \frac{\phi+1}{2\phi+1} = \frac{1}{\phi}$. Each leads to the equivalent equation $\phi^2 = \phi + 1$ (which is equivalent to the equation in 2c on the first page of the investigation.)



2. The TI-84 Plus C program is provided to create the figure readily and more aesthetically, which may be useful to show students at the end of the activity, especially if you then sketch the golden spiral on top of it.

Some of the functions used in the Shade command in the program are piecewise functions created using the 2nd [TEST] menu. For example, `(0<X and X<A)` is used to restrict the domain of the line $y = 1$ to values of x in the interval $0 < x < \phi$, since `A` $= \phi$.

Since `(0<X and X<A)` $= \begin{cases} 1, & 0 < x < \phi \\ 0, & \text{else} \end{cases}$

Then $y =$ `1/(0<X and X<A)` $= \begin{cases} 1, & 0 < x < \phi \\ \text{undefined}, & \text{else} \end{cases}$

Thus we have a command such as
`Shade(0,1/(0<X and X<A),0,A,1,1,MAGENTA)`.

The program to the right does not draw the spiral shown, but you can get this update on the Website http://users.ipfw.edu/lamaster/technology/ and send it to your TI-84 Plus C using TI-Connect.

It is illuminating to explore the width and length of each of the whirling rectangles, which have a fascinating pattern imbued with the aroma of Fibonacci. This is another exploration in itself, but rewarding to pursue.

```
NORMAL FLOAT AUTO REAL RADIAN MP
PROGRAM:GOLDEN
:Func:DetectAsymOff
:BackgroundOff:AxesOn
:-.6→Xmin:6→Xmax:1→Xscl
:-.1→Ymin:4→Ymax:1→Yscl
:FnOff
:GridLine
:"1/X"→Y1
:"X-1"→Y2
:GraphColor(1,BLUE)
:GraphStyle(1,2)
:GraphColor(2,RED)
:GraphStyle(2,2)
:(1+√(5))/2→A
:A-1→B
:Shade(0,1/(0<X and X<A),0
,A,1,1,MAGENTA)
:Line(0,0,1,0,LTBLUE)
:Line(1,0,1,1,LTBLUE)
:Line(1,1,0,1,LTBLUE)
:Line(0,1,0,0,LTBLUE)
:Shade(0,1/(0<X and X<1),0
,1,1,1,LTBLUE)
:Shade(0,B/(1<X and X<A),1
,A,1,1,GREEN)
:Shade(1/(0<X and X<A),(1+
A)/(0<X and X<A),0,A,1,1,R
ED)
:Shade(0,(1+A)/(A<X and X<
2A+1),A,2A+1,1,1,NAVY)
```

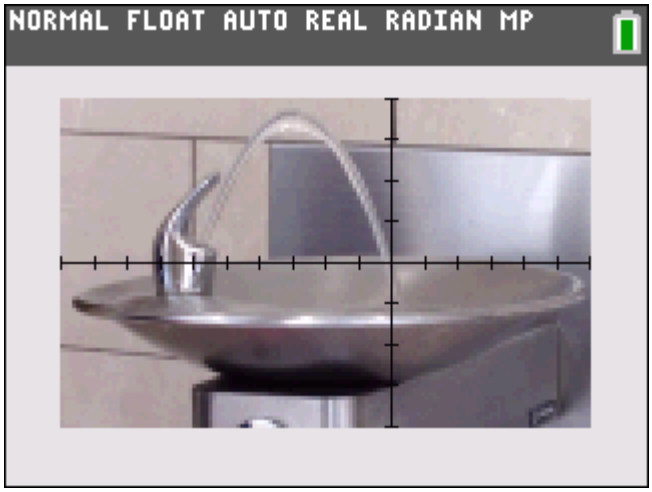
3. See the Web page by Weisstein, Eric W. called "Golden Rectangle" from *MathWorld*-- A Wolfram Web Resource http://mathworld.wolfram.com/GoldenRectangle.html

Investigation 3: At the Drinking Fountain

This activity provides opportunities to foster:
MP#4. Model with mathematics.
MP#5. Use appropriate tools strategically.
MP#7 Look for and make use of structure.

Lauren wants to overlay the graph of a parabola onto the photo of the drinking fountain.

She has many ways she can position the axes. One possibility is shown to the right.



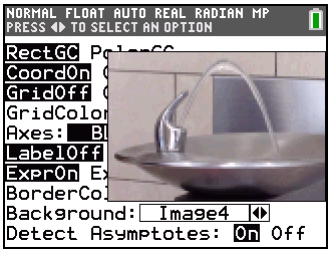
Assume Lauren wants the equation of her parabola to pass through the origin.

1. If each tick mark represents one unit, what might be a reasonable estimate for the other zero, to the nearest integer? _____

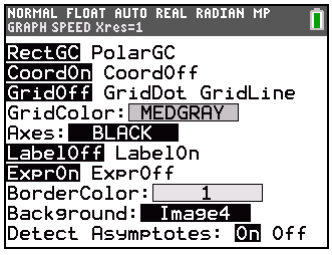
2. Using what you know about the symmetry of a parabola, estimate the coordinates of its vertex: (_____ , _____)

3. Given the two zeros of the parabola and one other point, use algebra to find its equation. Create an appropriate model of the parabolic stream of water.

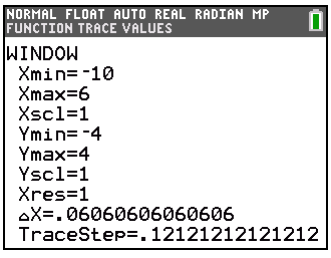
4. Press [2nd] [FORMAT] to access the Graph Format menu. For the Background, use the left or right arrow keys to scroll through the options. Image variables are shown, followed by choices for solid color backgrounds. Use built-in Image4.

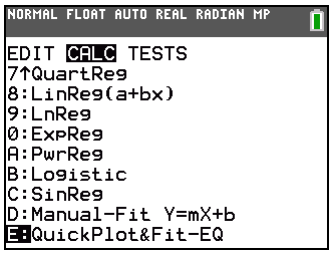


Once you have made your selection, you need only press the up or down arrow key and Image4 should be highlighted.



Press WINDOW and use the settings shown below.
Clear any functions in Y=. Turn off any STAT Plots.
Press STAT and use the right arrow to reach the **CALC** menu.



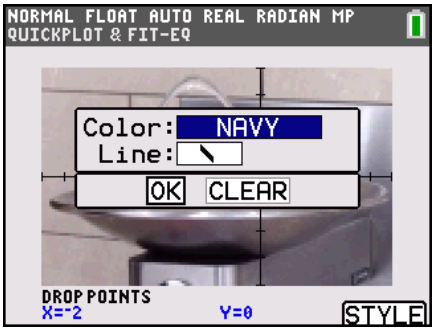


Scroll to the end of the menu to highlight **E:QuickPlot&Fit-EQ**. Press [ENTER].

5. Press the **STYLE** soft key.
   Use the left and right arrow keys to change the color of the drop points and the equation.
   (The *x*- and *y*-coordinates will also change to the selected color.)

   You can also change the line style.

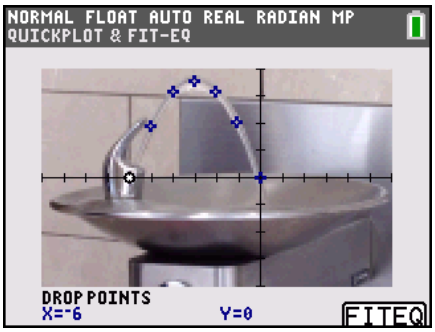
   Highlight **OK** and press [ENTER].



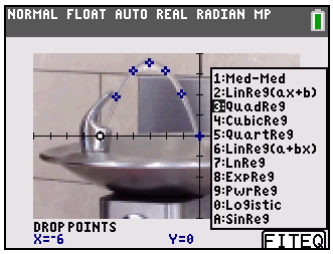
6. Move the cursor to the point (0, 0). Press enter to drop the point. Continue moving the cursor and dropping points on the graph.

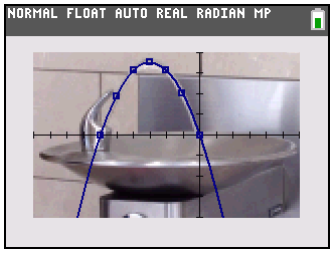
   (If you make an error, press [2nd] [QUIT])

   When you have dropped the desired number of points, press the **FITEQ** soft key.



7. Choose the regression model **3:QuadReg**.
   The equation of the regression model $y = ax^2 + bx + c$ is displayed and graphed.
   Follow the prompts to store the data in L1 and L2 and the equation in Y1.

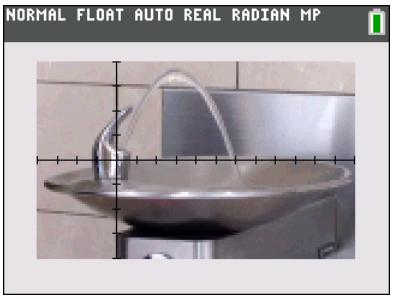




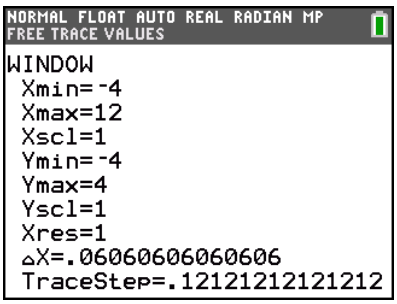
   a. The value of *c* in the regression model should be near 0. Why?

   b. How do the values of *a* and *b* in the quadratic regression equation compare with those of your model found analytically in Step 3?

8. Warren models the parabolic stream using the axes shown to the right. He uses the same values as Lauren for Ymin and Ymax, but uses Xmin = −4, Xmax = 12. Suppose he too uses integer values for the zeros of his parabola. Predict the following:
   a. How would his value of *a* in his quadratic equation compare with the value of *a* found in Lauren's model? Why?
   b. How would his value of *b* compare with the value of *b* found in Lauren's model? Why?



9. Find an equation for Warren's model using **QuickPlot&Fit-EQ**.
   Store the data in L_3 and L_4 and the equation in Y_2.
   Graph the equations for Lauren's and Warren's model in the same viewing window shown. How are these graphs related?



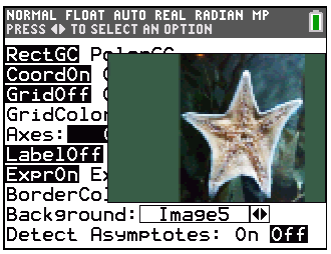
Extension

1. Create a viewing window which positions the vertex on the *y*-axis.
2. How does this affect the equation of the model?

Investigation 4: A Family of Starfish

This activity is inspired by the stretching and shrinking of MugWumps in the Grade 7 Connected Mathematics Project (**Michigan State University and Pearson Education, Inc.)** series.

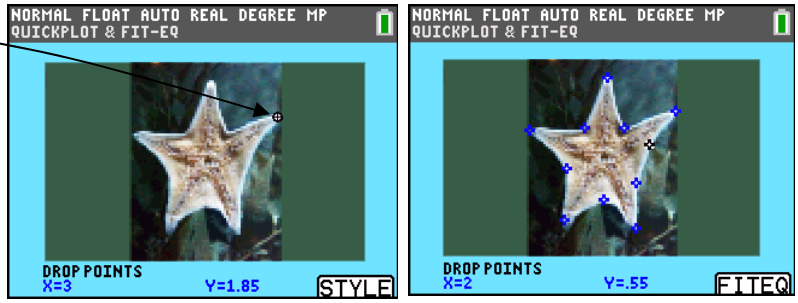
1. Press [2nd] [FORMAT] to access the Graph Format menu.
   - Turn the Axes Off.
   - Use BorderColor 3.
   - For the Background, use built-in Image5.



2. Clear any functions in Y=. Turn off any STAT Plots. Press ZOOM and use **4:ZDecimal**.

3. Press STAT and use the right arrow to reach the **CALC** menu. Scroll to the end of the menu to highlight **E:QuickPlot&Fit-EQ**. Press [ENTER].

4. Drop points along the border of the starfish which will later be connected by line segments through the STAT PLOT menu.

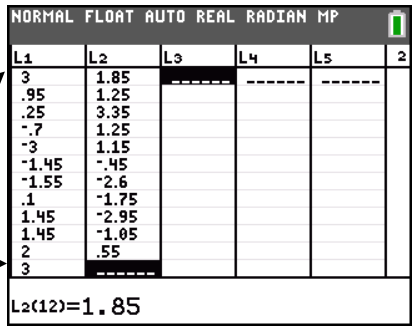


   Some Tips:
   - *Drop points along the path in order.*
   - *The default color (blue) can be used. You can change the color later.*
   - *If you make a mistake, press* [2nd] [QUIT], *followed by* [2nd] [DRAW] **1:ClrDraw** *and press* [ENTER]. *Then start again.*

5. Once you have dropped the points you want, press [ALPHA] [F5] to FITEQ. The model will be drawn on the screen. Press [ALPHA] [F5] to STORE. Use L1 and L2. Select OK and press [ENTER].

6. Press STAT **1:Edit**. The first point and the last point should be the same.

   If not, press [2nd] [INS] and add the last point so it is the same as the first.

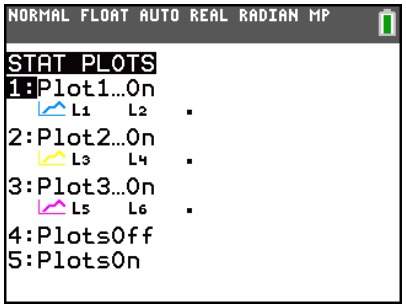


New with TI-84 Plus C:
You can readily distinguish the *STAT Editor* (monochrome) from the *Table Editor* (color).

7. On the home screen
   - create L3 and L4 using `1.2L1→L3` and `1.2L2→L4`
   - create L5 and L6 using `.5L1→L5` and `.5L2→L6`

   Explore the data in the STAT Editor. What happens to the coordinates of the original starfish?

8. Set up PLOT1, PLOT 2, and PLOT3 using connected lineplots and thick-dot. (Colors shown are LTBLUE, YELLOW, and MAGENTA.) Describe what happens to the shape of the original starfish.



How to Transfer an Image to your TI-84 Plus C

What You Need to Begin

- TI-84Plus Color Silver Edition.
- Computer.
- The latest version of TI Connect™ software installed on your machine. Go to **education.ti.com**
  If you plug in the cable before installing the TI Connect™ software, Windows® may assign an incorrect driver for the cable. Install TI Connect FIRST!!!
- TI Connectivity Cable.
  These are available for purchase by entering "TI Connectivity Cable" in your favorite search engine.
  (You will have a slower transfer with the silver cable.)

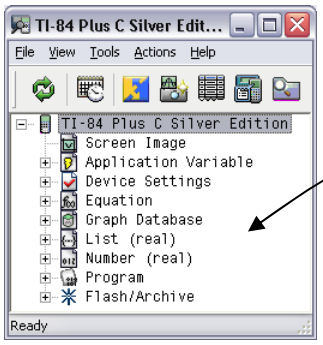
USB for Windows® or Mac® (silver)

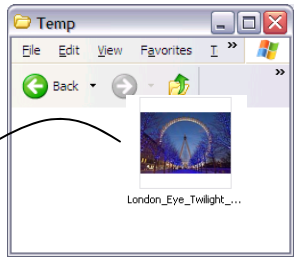
Standard Mini-A to Mini-B USB Cable for Windows® or Mac®

- Your favorite image.
  We will use a photo from the Wikipedia Commons.
  Permission is granted to copy, distribute and/or modify this image.
  If you would like to use this photo, download it from
  http://en.wikipedia.org/wiki/File: London_Eye_Twilight_April_2006.jpg

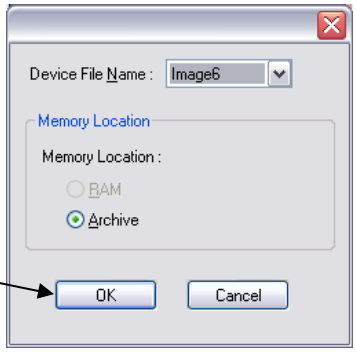
Use TI-Connect to Drag and Drop

1. Start the TI Connect™ software that has already been installed on your machine.
2. Connect the TI-84Plus Color Silver Edition to the computer with the TI Connectivity Cable
3. Click on **TI Device Explorer** on TI Connect to open the DeviceExplorer window.
4. Drag the digital image into the DeviceExplorer window.





5. You will be prompted for a name for the Image Var (Image0 through Image9).
   Click OK when done.



Alternatively, you can transfer images from one unit to another.

Now use the Graph Format menu to display your image as a background.
Change your mode to Radians and Parametric,
press Y= to enter the equations shown
with the animated path line style,
and press **ZOOM 4:ZDecimal** to ride the London Eye.