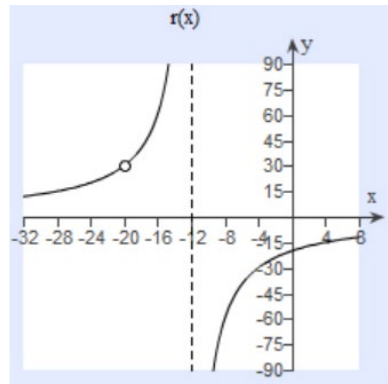
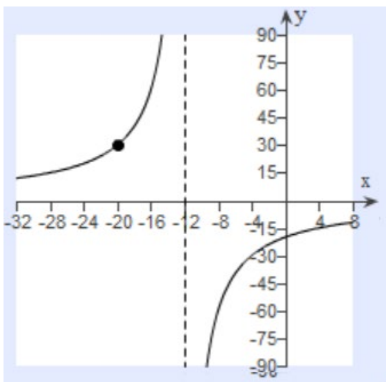
Find a formula for a rational function $r(x)$ in the form $y = \frac{p(x)}{q(x)}$ where is the $q(x)$ is the lowest polynomial degree possible, with the following properties.

1. $\lim_{x \to -12^-} r(x) = \infty$ and $\lim_{x \to -12^+} r(x) = -\infty$
2. $\lim_{x \to -20} r(x) = 30$
3. $r(-20)$ is undefined.
4. $\lim_{x \to \pm\infty} r(x) = 0$

The above indicates that we have a vertical asymptote at $x = -12$ and a hole at $(-20, 30)$.
To the right is a graph with these properties

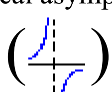


Step 1: Write a formula of the function without the hole.



Step 2: Incorporate the *location* of the vertical asymptote into the formula. This corresponds to the *zero* of the factor.
A vertical asymptote occurs at $x = h$ if the denominator has a factor $(x - h)$ and the numerator does not.

Step 3: Incorporate the *behavior* near the vertical asymptote into the formula. This corresponds to the *power* of the factor.

Near the vertical asymptote $x = -12$, the shape  of the graph indicates the power of the factor is **odd**.
Mathematically, this is written that from the left ($\lim_{x \to -12^-} r(x) = \infty$) and from the right ($\lim_{x \to -12^+} r(x) = -\infty$)

Because $q(x)$ must be the smallest degree possible, the degree of the denominator must be **1**.

So this means the formula is of the form $y = \frac{k}{x+12}$ (and in fact we expect k to be negative based on the graph.)

Step 4: Find the formula by plugging in the point $x = -20$, $y = 30$ and solve for k.

$$y = \frac{k}{x+12}$$
$$30 = \frac{k}{-20+12}$$
$$30 = \frac{k}{-8}$$
$$k = 30 \cdot (-8)$$
$$= -240$$

Thus $y = \frac{-240}{x+12}$. (Check with a grapher that it passes through the point $x = -20$, $y = 30$ using a table feature.)

Step 5: Modify the function so it has a hole at $x = -20$. Using common factors, we have $r(x) = \frac{-240(x+20)}{(x+12)(x+20)}$.