LET'S PLAY CALC-PARDY!!

http://www.magicnet.net/~itms/jeopardy/index.htm

Calc-pardy

Chain Gang	We're Related	Optimus Prime	Vital Signs	Critical Thinking
Q $200	Q $200	Q $200	Q $200	Q $200
Q $400	Q $400	Q $400	Q $400	Q $400
Q $600	Q $600	Q $600	Q $600	Q $600
Q $800	Q $800	Q $800	Q $800	Q $800
Q $1000	Q $1000	Q $1000	Q $1000	Q $1000

Final Jeopardy

$200 Answer from Chain Gang

$$y = \ln x^9 = 9\ln x$$

$$y' = 9 \cdot \frac{d}{dx}\ln x$$

$$= 9 \cdot \frac{1}{x}$$

$$= \frac{9}{x}$$

$400 Answer from Chain Gang

$$y = (5x^{10} + 10)^{20}$$

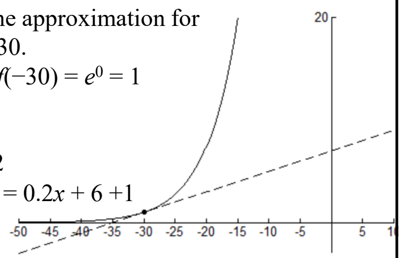
$$y' = 20(5x^{10} + 10)^{19} \cdot (50x^9)$$

$$= 1000x^9(5x^{10} + 10)^{19}$$

\$600 Answer from Chain Gang

$L(x)$ is the tangent line approximation for $f(x) = e^{0.2x+6}$ at $x = -30$.

If $f(x) = e^{0.2x+6}$ then $f(-30) = e^0 = 1$
and $f'(x) = 0.2e^{0.2x+6}$
so
$f'(-30) = 0.2e^0 = 0.2$
$L(x) = 0.2(x+30) + 1 = 0.2x + 6 + 1$
$= \mathbf{0.2x + 7}$



$L(-33) = 0.4$ and $f(-33) \approx \mathbf{0.548812}$
$L(-3) = 6.4$ and $f(-3) \approx \mathbf{221.4}$
So $L(-33)$ is a better approximation of the function value.

The graph helps you see this by inspection.

\$800 Answer from Chain Gang

$$y = \frac{e^{2x}}{e^x - 4}$$

$$y' = \frac{(e^x-4)\cdot\frac{d}{dx}e^{2x} - e^{2x}\cdot\frac{d}{dx}(e^x-4)}{(e^x-4)^2} = \frac{(e^x-4)\cdot 2e^{2x} - e^{2x}\cdot e^x}{(e^x-4)^2}$$

$$= \frac{2e^{2x}\cdot e^x - 8e^{2x} - e^{2x}\cdot e^x}{(e^x-4)^2}$$

$$= \frac{2e^{3x} - 8e^{2x} - e^{3x}}{(e^x-4)^2}$$

$$= \frac{e^{3x} - 8e^{2x}}{(e^x-4)^2}$$

$$= \frac{e^{2x}(e^x-8)}{(e^x-4)^2}$$

\$1000 Answer from Chain Gang

$$y = \tan^{-1}(e^x)$$

$$= \tan^{-1}(u) \rightarrow u = e^x, \quad \frac{du}{dx} = e^x$$

$$y' = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

$$= \frac{1}{1+(e^x)^2}\cdot e^x$$

$$= \frac{e^x}{1+e^{2x}}$$

\$200 Answer from We're Related

If a circle's radius increases at 6 cm / s, find the rate the area increases when the radius is 10 cm.

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\cdot\frac{dr}{dt}$$

$$= 2\pi 10\cdot 6$$

$$= 120\pi \text{ cm}^2/\text{s}$$

$400 Answer from We're Related

If the sides of a cube increase at 6 cm / s, find the rate the volume increases when the side length is 10 cm.

$$V = x^3$$

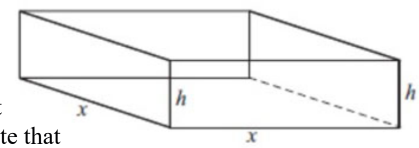
$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$= 3 \cdot 10^2 \cdot 6$$

$$= 1800\,\text{cm}^3/\text{s}$$

$600 Answer from We're Related

If the height h increases at 8 cm / s, and the base x is fixed at 2 cm, find the rate that the volume increases.



$$V = 2^2 h$$

$$V = 4h$$

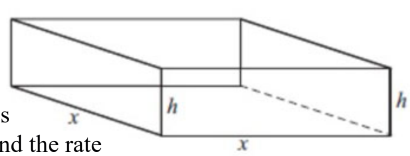
$$\frac{dV}{dt} = 4 \cdot \frac{dh}{dt}$$

$$= 4 \cdot 8$$

$$= 32\,\text{cm}^3/\text{s}$$

$800 Answer from We're Related

If the volume V increases at 5 cm^3 / s, and the height h is fixed at 10 cm, find the rate that the base x increases when the base $x = 2$.



$$V = x^2 h = x^2 \cdot 10 = 10x^2$$

$$\frac{dV}{dt} = 20x \cdot \frac{dx}{dt}$$

$$\frac{dV}{dt} = 20x \cdot \frac{dx}{dt}$$

$$5 = 20x \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{5}{20x} = \frac{1}{4x}$$

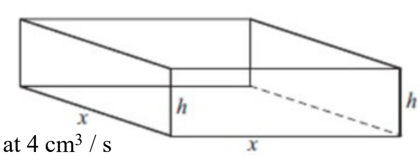
$$\frac{dx}{dt} = \frac{1}{4x}$$

$$= \frac{1}{4 \cdot 2}\,\text{cm/s}$$

$$= \frac{1}{8}\,\text{cm/s}$$

$1000 Answer from We're Related

A rectangular tank is filled with 400 cm^3 of water. If the volume decreases at 4 cm^3 / s and the base x is fixed at 2 cm, write h as a function of t.



$$V = 400 - 4t$$

$$\frac{dV}{dt} = -4$$

$$V = x^2 h = 2^2 h$$

$$= 4h$$

$$400 - 4t = 4h$$

$$h = 100 - t$$

$$\frac{dh}{dt} = -1 \ \text{cm/s}$$

OR

$$V = 4h$$

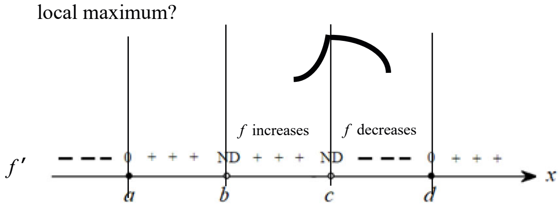
$$\frac{dV}{dt} = 4 \cdot \frac{dh}{dt}$$

$$-4 = 4 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = -1\,\text{cm/s}$$

$200 Answer from Optimus Prime

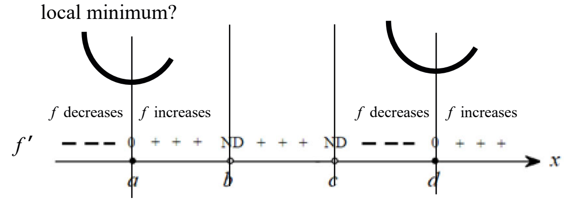
The signs of f' are shown. For what value(s) does f have a local maximum?



f has a local maximum at $x = c$.

$400 Answer from Optimus Prime

The signs of f' are shown. For what value(s) does f has a local minimum?

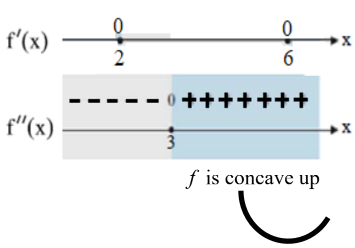


f has a local minimum at $x = a$ and $x = d$.

$600 Answer from Optimus Prime

The signs of f' and f'' are shown. For what value(s) does f have a local minimum?

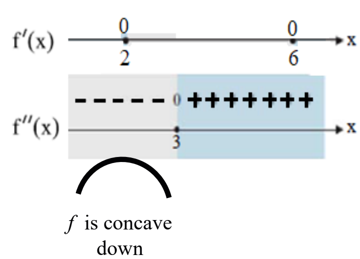
f has a local minimum at $x = 6$.



$800 Answer from Optimus Prime

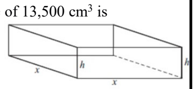
The signs of f' and f'' are shown. For what value(s) does f have a local minimum?

f has a local maximum at $x = 2$.



\$1000 Answer from Optimus Prime

A rectangular tank with a square base, an open top, and volume of 13,500 cm³ is to be constructed of sheet steel. The tank with the minimum surface area has a square base with a side length of ? and height of ?



$V = 13500$

$x^2 h = 13500$

$h = \frac{13500}{x^2}$

$S = 4x \cdot h + x^2$

$S = 4x \cdot \frac{13500}{x^2} + x^2$

$S = \frac{54000}{x} + x^2$

$S = 54000x^{-1} + x^2$

$S' = -54000x^{-2} + 2x$

$S' = -\frac{54000}{x^2} + 2x$

$-\frac{54000}{x^2} + 2x = 0$

$2x = \frac{54000}{x^2}$

$x^3 = \frac{54000}{2}$

$x^3 = 27000$

$x = 30$

$h = \frac{54000}{x^2}$

$h = \frac{54000}{30^2}$

$h = 15$

```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
\Y1=54000/X+X²
\Y2=13500/X²
\Y3=-54000/X²+2X
```

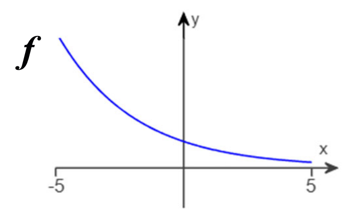
base X	S Y1	height Y2	S' Y3
27	2729	18.519	-20.07
28	2712.6	17.219	-12.88
29	2703.1	16.052	-6.209
30	2700	15	0
31	2702.9	14.048	5.8085
32	2711.5	13.184	11.266
33	2725.4	12.397	16.413
34	2744.2	11.678	21.287
35	2767.9	11.02	25.918
36	2796	10.417	30.333
37	2828.5	9.8612	34.555

a side length of 30 and height of 15

\$200 Answer from Vital Signs

Sign of *f ″*

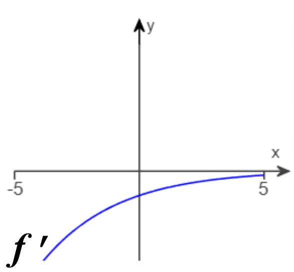
Since *f* is concave up, *f ″* is positive.



\$400 Answer from Vital Signs

Sign of *f ″*

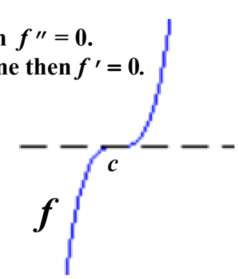
Since *f ′* is increasing, *f ″* is positive



\$600 Answer from Vital Signs

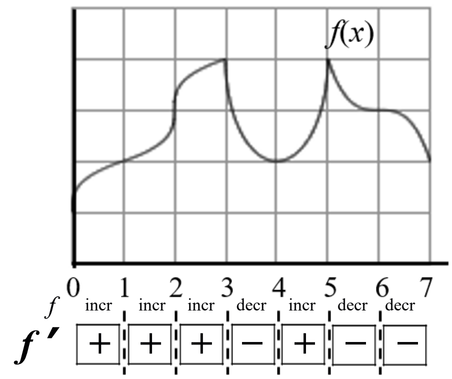
Sketch a graph of *f* which has a value *x* = *c* where *f ′* and *f ″* are both 0

If *f* has an inflection point then *f ″* = 0.
If *f* has a horizontal tangent line then *f ′* = 0.



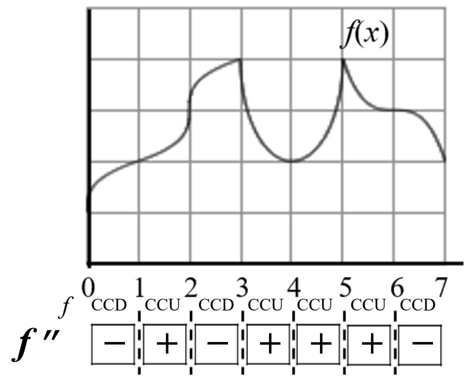
$800 Answer from Vital Signs

Put + or – in each box



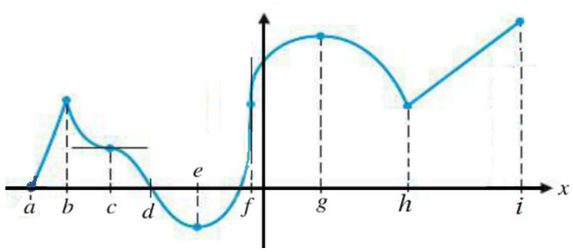
$1000 Answer from Vital Signs

Put + or – in each box



$200 Answer from Critical Thinking

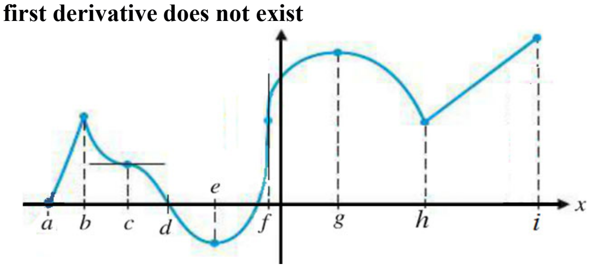
List all the critical values of the function



The function has critical values at $x = b, c, e, f, g, h$

$400 Answer from Critical Thinking

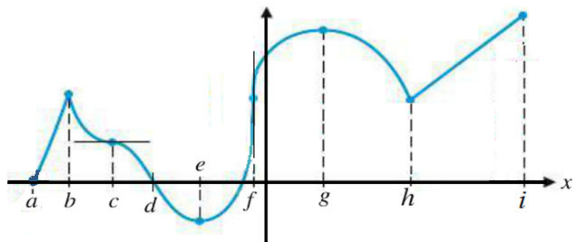
List all the critical values of the function where the first derivative does not exist



The first derivative does not exist at $x = b, f,$ and h

$600 Answer from Critical Thinking

Which critical values correspond to **neither** local minima or local maxima?

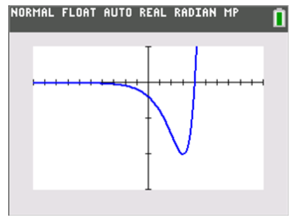


At $x = c$ and $x = f$

$800 Answer from Critical Thinking

$f(x) = e^x(x-4)$ has critical value at $x = 3$.

Determine the sign of $f''(3)$.



At $x = 3$ the graph has a minimum and f is concave up so $f''(3)$ is positive.

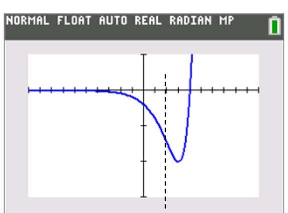
$1000 Answer from Critical Thinking

$f(x) = e^x(x-4)$ Find where $f''(x) = 0$.

$f(x) = xe^x - 4e^x$

$f'(x) = (x \cdot \frac{d}{dx}e^x + e^x \cdot \frac{dx}{dx}) - 4e^x = xe^x + e^x - 4e^x = xe^x - 3e^x$

$f''(x) = \frac{d}{dx}xe^x - \frac{d}{dx}3e^x = (xe^x + e^x) - 3e^x = xe^x - 2e^x = e^x(x-2)$



At $x = 2$ the graph changes concavity and $f''(2) = 0$.

Final Jeopardy Answer

Find $g'(7)$

$f'(x) = 3x^2 + \frac{1}{2}$

$f'(2) = 3 \cdot 2^2 + \frac{1}{2} = 12.5$ or $\frac{25}{2}$

$g'(7) = \frac{1}{f'(2)} = \frac{1}{12.5}$ or $\frac{2}{25}$ or 0.08