Section 4.7: L'Hôpital's Rule or L'Hôspital's Rule

1. For the function $f(x) = x^2$, what is happening to the y-values as x approaches infinity?

2. For the function $g(x) = \ln x$, what is happening to the y-values as x approaches infinity?

3. Which function, $f(x)$ or $g(x)$ is growing at a faster rate? How do you know?

4. Evaluate $\lim_{x\to\infty} \frac{\ln x}{x^2}$. Explain your thinking.

5. Let $f(x) = x^2 + 2x - 15$ and $g(x) = x - 3$.
    a. Evaluate $f(3)$ and $g(3)$.

    b. Why does direct substitution not work to evaluate $\lim_{x\to 3} \frac{f(x)}{g(x)}$?

    c. Use the factoring method to evaluate $\lim_{x\to 3} \frac{f(x)}{g(x)}$

6. Now let's look at this limit from a different perspective.
    a. How do the *values* of $f(x)$ and $g(x)$ at $x = 3$ compare?

    b. Which function, $f(x)$ or $g(x)$, is changing at a faster *rate* at $x = 3$? How do you know?

7. Evaluate $\lim_{x\to 3} \frac{f'(x)}{g'(x)}$. What do you notice?

8. When do you think comparing growth rates is a helpful strategy for evaluating limits?

Important Ideas:

Check Your Understanding! Use L'Hôpital's Rule when it is convenient and applicable.

1. $\lim_{x\to 0}\frac{k\sin ax}{bx}$ 2. $\lim_{x\to 0}\frac{e^x-1}{\sin x}$ 3. $\lim_{x\to -7}\frac{x^2+10x+21}{x^2-49}$ 4. $\lim_{x\to 0}\frac{4x^3-6x^2}{-5x^2}$ 5. $\lim_{x\to\infty}\frac{x^3\ln x}{x^5}$

6. $\lim_{x\to\infty}\frac{x^{999}}{x^{1000}}$ 7. $\lim_{x\to 0}x^3\cot x$ 8. $\lim_{x\to\infty}\frac{e^{10/\sqrt{x}}-1}{\frac{10}{\sqrt{x}}}$ 9. $\lim_{x\to\infty}(1+\frac{r}{x})^x$ 10. $\lim_{x\to 0}(1+2x)^{5/x}$

11. Compare $y = \ln x$, $y = e^x$, and $y = x^n$ for positive n. Which grows fastest as $x \to \infty$?