## Bad Boss

Kilo Ren has just taken over as manager of a struggling droid production company. The profit $P(t)=t^{3}-90 t^{2}+1500 t+3000$ at day $t$ seems to go well at in the first ten days, but then morale erodes and profits start to tank. Complete the table.
Recall $\int_{0}^{x} P^{\prime}(t) d t=P(x)-P(0)$. We can also write $\mathrm{P}(\mathrm{x})-\mathrm{P}(0)=\left.\mathrm{P}(\mathrm{t})\right|_{0} ^{x}{ }_{0}^{x} \quad$ "bar notation"


| $t$ | $P^{\prime}(t)$, <br> days | $P(t)$, <br> $\$$ <br> $\$$ | Area under $P^{\prime}(t)$ <br> from $x=0$ to $x=t$ |
| ---: | ---: | ---: | :--- |
| 0 | 1500 | 3000 |  |
| 10 | 0 | 10000 |  |
| 20 | -900 | 5000 |  |
| 30 | -1200 | -6000 |  |
| 40 | -900 | -17000 |  |
| 50 | 0 | -22000 |  |
| 60 | 1500 | -15000 |  |

1. Report $P(0)$, using appropriate units, and interpret.
2. Consider the function $f(x)$ that gives the area under $\mathrm{P}^{\prime}(\mathrm{t})$ from $\mathrm{t}=0$ to $\mathrm{t}=\mathrm{x}$, where x is the number of days that Kilo Ren has been manager.
a. Write $f(x)$ as an integral. Interpret in the context of the situation.
b. Use your table to sketch $f(x)$ on the graph above. How are $f$ and $P$ related graphically?
c. For what values of $x$ is the $f(x)$, the area under $P^{\prime}(t)$ on the interval [ $\left.0, x\right]$, increasing? What is true about $P$ for these values? What is true about $f$ for these values?
d. For what values of $x$ is $f(x)$, the area under $P^{\prime}(t)$ on the interval $[0, x]$, decreasing? What is true about $P$ for these values? What is true about $f$ for these values?
e. When is $f(x)$ a minimum? When is $f(x)$ a maximum? How do you know?
f. When is $f(x)$ changing the fastest? How do you know?
g. Report f ' 30 ).
3. What is the same about $f$ and $P$ ? What is different?
4. Construct formulas for $f(x)$ and $f^{\prime}(x)$ without integrals.
