Equilibrium problems for infinite dimensional vector potentials with external fields

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The lecture deals with a minimal energy problem in the presence of an external field $\mathbf{f} = (f_i)_{i\in I}$ over noncompact classes of infinite dimensional vector measures $\mu = (\mu^i)_{i\in I}$ in a locally compact space. The components μ^i are positive measures (charges) normalized by $\int g_i\,d\mu^i = a_i$ (where a_i and g_i are given) and supported by given closed sets A_i with the sign $+1$ or -1 prescribed such that $A_i \cap A_j = \varnothing$ whenever $\operatorname{sign} A_i \neq \operatorname{sign} A_j$, and the law of interaction of μ^i, $i \in I$, is determined by the interaction matrix $\bigl(\operatorname{sign} A_i \operatorname{sign} A_j\bigr)_{i,j\in I}$. For all positive definite kernels satisfying Fuglede's condition of consistency between the vague (=weak*) and strong topologies, sufficient conditions for the existence of equilibrium measures are established and properties of their uniqueness, vague compactness, and continuity under exhaustion of A_i by compact K_i are studied. Sharpness of the statement on the existence of equilibrium measures is discussed by providing examples of non-solvability. We also obtain variational inequalities for the $\mathbf{f}$-weighted equilibrium potentials, single out their characteristic properties, and analyze continuity of the equilibrium constants. Such results are new even for classical kernels in $\mathbb{R}^n$, which is important in applications.