

THE DEPARTMENT OF MATHEMATICAL SCIENCES

Purdue University Fort Wayne

is pleased to present

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## How “Sharp” is the Łojasiewicz Gradient Inequality?

### Abstract

The (famous) Łojasiewicz gradient inequality says that given a real analytic function  $f$  such that  $f(x_0) = \nabla f(x_0) = 0$ , there exists a number  $\beta \in (0, 1)$  such that

$$|\nabla f(x)| \geq c |f(x)|^\beta$$

for  $x$  in a neighborhood of  $x_0$ . The smallest such  $\beta$  is called the Łojasiewicz exponent of  $f$  at  $x_0$ . Recently, Feehan gave a geometric proof of the above statement using Hironaka’s theory of resolution of singularities. Moreover he showed that the range of the Łojasiewicz exponent is in fact  $[\frac{1}{2}, 1)$ , and he proved that the Łojasiewicz exponent is  $\frac{1}{2}$  if and only if  $f$  is a Morse-Bott function (a generalized version of a Morse function). We introduce a notion measuring how sharp the Łojasiewicz inequality is in an integral sense, using a general blow-up phenomenon recently discovered by Pan. We will discuss examples illustrating the potential usefulness of this point of view. Joint work with Yifei Pan.

11:00 – Noon, Monday, November 25, 2019. Location: Kettler 218

<http://www.pfw.edu/departments/coas/depts/math/news/seminars.html>