ABSTRACTS OF TALKS PRESENTED TO THE MIDWEST WORKSHOP ON ASYMPTOTIC ANALYSIS / ANALYSIS MINI-SYMPOSIUM AT IPFW

For 2014, on the occasion of IPFW's 50^{th} Anniversary, the fourth annual Mini-Symposium on Analysis is organized concurrently with the Midwestern Workshop on Asymptotic Analysis.

Friday/Saturday, Sept. 19/20, 2014.

Sponsored by:

- IPFW Department of Mathematical Sciences
- IPFW Office of Research, Engagement, and Sponsored Programs
- Simons Foundation Collaboration Grant
- IPFW Pippert Science Research Scholar grants

1. Abstracts of Talks

Presenter: Alexander I. Aptekarev, Keldysh Institute of Applied Mathematics, Russian Academy of Sciences

Joint work with: A. Draux, V. A. Kalyagin and D. N. Tulyakov.

Asymptotics of sharp constants of Markov-Bernstein inequalities in integral norm with Jacobi weight

The classical A. Markov inequality establishes a relation between the maximum modulus or the $L^\infty([-1,1])$ norm of a polynomial Q_n and of its derivative: $\|Q_n'\| \leqslant M_n n^2 \|Q_n\|$, where the constant $M_n = 1$ is sharp. The limiting behavior of the sharp constants M_n for this inequality, considered in the space $L^2\left([-1,1], w^{(\alpha,\beta)}\right)$ with respect to the classical Jacobi weight $w^{(\alpha,\beta)}(x) := (1-x)^\alpha (x+1)^\beta$, is studied. We prove that, under a technical condition $|\alpha - \beta| < 4$, the limit is $\lim_{n\to\infty} M_n = 1/(2j_\nu)$ where j_ν is the smallest zero of the Bessel function $J_\nu(x)$ and $2\nu = \min(\alpha,\beta) - 1$. Recently, V. Totik, based on our result, has removed this technical condition on the parameters of the Jacobi weights.

Presenter: Pavel Bleher, IUPUI

Joint work with: K. Liechty, DePaul University.

Six-vertex model with partial domain wall boundary conditions: ferroelectric phase

We present an exact solution to the large N limit of the six-vertex model with partial domain wall boundary conditions in the ferroelectric phase. The solution consists of two steps. In the first step we derive a formula for the partition function involving the determinant of a matrix of mixed Vandermonde/Hankel type. This determinant can be expressed in terms of a system of discrete orthogonal polynomials, which can then be evaluated asymptotically by comparison with the Meixner polynomials.

Presenter: Tom Bloom, University of Toronto

Large deviation for outlying coordinates in beta ensembles

For Y a subset of the complex plane, a β ensemble is a sequence of probability measures $Prob_{n,\beta,Q}$ on Y^n for $n = 1, 2, \ldots$ depending on a positive real parameter β and a real-valued continuous function Q on Y. We consider the associated sequence of probability measures on Y where the probability of a subset W of Y is given by the probability that at least one coordinate of Y^n belongs to W. With appropriate restrictions on Y, Q we prove a large deviation principle for this sequence of probability measures. This extends a result of Borot-Guionnet to subsets of the complex plane and to β ensembles defined with measures using a Bernstein-Markov condition.

Presenter: Dusty Grundmeier, Ball State University

Asymptotic properties of group-invariant CR mappings

In this talk we will examine asymptotic properties of a family of polynomials that naturally arises in CR geometry. In particular we will show how these polynomials are intimately related to Chebyshev polynomials.

Presenter: Doug Hardin, Vanderbilt University

Joint work with: P. Boyvalenkov, IMI, Sofia; P. Dragnev, IPFW; E. Saff, Vanderbilt University; M. Stoyanova, Sofia University.

Universal lower bounds for potential energy of spherical codes

Based upon the works of Delsarte-Goethals-Seidel, Levenshtein, Yudin, and Cohn-Kumar we derive universal lower bounds for the potential energy of spherical codes, that are optimal (in the framework of the standard linear programming approach) over a certain class of polynomial potentials whose degrees are upper bounded via a familiar formula for spherical designs. We classify when improvements are possible employing polynomials of higher degree. Our bounds are universal in the sense of Cohn and Kumar; i.e., they apply whenever the potential is given by an absolutely monotone function of the inner product between pairs of points.

Presenter: Alexander Its, IUPUI
Joint work with: T. Bothner, P. Deift, and I. Krasovsky.
On the transitional asymptotics of the sine-kernel determinant
We study the determinant $\det(I - \gamma K_s)$, $0 < \gamma < 1$, of the Fredholm operator K_s acting on the interval $(-1, 1)$ with kernel $K_s(\lambda, \mu) = \frac{\sin(s(\lambda-\mu))}{\pi(\lambda-\mu)}$. This determinant represents one of the fundamental distribution functions of random matrix theory. We evaluate, in terms of elliptic theta-functions, the double scaling limit of $\det(I - \gamma K_s)$ as $s \to \infty$ and $\gamma \uparrow 1$, in the region $cs^{-\epsilon} \leq -\frac{1}{2s}\ln(1-\gamma) \leq 1 - \delta$, for any fixed $0 < \delta < 1$. This problem was first considered by Dyson in 1995.

Presenter: Alexander Izzo, Bowling Green State University
Joint work with: H. Samuelsson Kalm, E. Stout, and E. F. Wold.
Manifolds with polynomially convex hull without analytic structure
It was once hoped that whenever a compact set in complex Euclidean space has a nontrivial polynomially convex hull, there must be analytic structure in the hull. This hope was dashed by a counterexample given by Stolzenberg in 1963. I will present recent joint work with Samuelsson Kalm and Wold showing that every smooth manifold of dimension at least three can be smoothly embedded in some complex Euclidean space so as to have hull without analytic structure and present current work with Stout extending this to two dimensional manifolds. (It is well known that a smoothly embedded one dimensional manifold never has hull without analytic structure.)

Presenter: Greg Knese, Washington University
Joint work with: Beneteau, Kosinski, Liaw, Seco, and Sola.
Cyclic polynomials in two variables
A vector is cyclic for an operator or family of commuting operators if the closed invariant subspace it generates is the whole Hilbert space. A famous result of Smirnov and Beurling says that the cyclic vectors for the shift operator on the Hardy space on the disk are exactly the outer functions. Generalizing this result to more dimensions and in particular to polydisks is well-motivated by the fact that characterizing cyclic vectors for the Hardy space on the infinite polydisk is closely related to Nyman's dilation completeness problem, which is known to be equivalent to the Riemann hypothesis. In this talk we confine ourselves to two variables and we completely characterize the cyclic *polynomials* for the shift operators on a range of Hilbert spaces of analytic functions on the bidisk which include the Hardy space and the Dirichlet space. The answer depends on the size and nature of the zero set of the polynomials on the distinguished boundary of the bidisk.

Presenter: László Lempert, Purdue University West Lafayette
Noncommutative potential theory
Ordinary potential theory is concerned with (pluri)subharmonic functions in the complex plane (or on higher dimensional real or complex manifolds). These functions can also be thought of as defining hermitian metrics on line bundles. Noncommutativity enters when one passes to holomorphic vector bundles with fibers of dimension > 1 and hermitian metrics on them. Such hermitian metrics locally can be represented by self adjoint matrix functions, and taking the curvature of the metric is analogous to applying the Laplacian to a scalar valued function. In the talk I will discuss properties of positively/negatively curved metrics, i.e. matrix functions, that generalize properties of (pluri)sub- and superharmonic functions.

Presenter: Andrei Martínez-Finkelshtein, Universidad de Almería
Joint work with: A. Aptekarev and G. López-Lagomasino.
Weak and strong asymptotics for the Pollaczek multiple orthogonal polynomials
Pollaczek multiple orthogonal polynomials are type II Hermite-Padé polynomials orthogonal with respect to two simple measures supported on the positive semi-axis. These measures form a so-called Nikishin pair, with the feature that one of its generators is purely discrete. It is known that the large-degree asymptotics of such polynomials is governed by the solution of a vector equilibrium problem, which was previously computed by V. Sorokin. For the strong asymptotics we use the Riemann-Hilbert characterization of the Hermite-Padé polynomials and the corresponding non-linear steepest descent method. We discuss some of the main ingredients of this analysis and the asymptotic results obtained by this method.

Presenter: Yifei Pan, IPFW
On flat solutions of $\bar{\partial}$-equation in any dimension
We construct a smooth function f that is flat at the origin, and is such that $\bar{\partial} u = f$ has no flat solutions.

Presenter: E. B. Saff, Vanderbilt University
Zeros of asymptotically extremal polynomials
We describe some simple sufficient geometric conditions on a compact set E of the plane under which the normalized counting measures of the zeros of any asymptotically extremal sequence of polynomials necessarily converges in the weak-star topology to the equilibrium measure for E. The question of existence of "electrostatic skeletons" for compact sets E arises naturally in the context of such asymptotic problems.

Presenter: B. Simanek, Vanderbilt University

Orthogonal polynomials on polynomial lemniscates

When attempting to generalize results from orthogonal polynomials on the unit circle to more general settings, a natural case to consider is that when the measure of orthogonality is concentrated on a region whose boundary is defined by the level set of a polynomial. In this talk, we will explain why this is the case and explore some recent results on this topic. One of our main results is a set of conditions on the entries of the Bergman Shift matrix that is equivalent to the measure being concentrated (in an appropriate sense) near the boundary of a polynomial lemniscate.

Presenter: Yuan Zhang, IPFW

Joint work with: X. Huang, Rutgers University.

CR transversality of holomorphic maps into hyperquadrics

In this talk, we discuss CR transversality of holomorphic maps between CR hypersurfaces. Let M_ℓ be a smooth Levi-nondegenerate hypersurface of signature ℓ in $\mathbf{C}^n$ with $n \geq 3$, and write H_ℓ^N for the standard hyperquadric of the same signature in $\mathbf{C}^N$ with $N - n < \frac{n-1}{2}$. Let F be a holomorphic map sending M_ℓ into H_ℓ^N. Assume F does not send a neighborhood of M_ℓ in $\mathbf{C}^n$ into H_ℓ^N. We show that F is necessarily CR transverse to M_ℓ at any point.