Abstract for talk by Doug Weakley in Discrete Math Seminar
4:30 p.m. on Wednesday, September 29, in Kettler 119

Is every C^∞-word recurrent?

The sequence $K = 12211212212211211221211212211...$ given by W. Kolakoski in 1965 can be described as an infinite sequence of 1's and 2's that begins with 1 and has the property that the length of the jth run of like symbols is equal to the jth symbol.

Question. What are the finite subwords of K?

Definitions. A finite word W of 1's and 2's in which neither 111 nor 222 occurs is *differentiable*, and its derivative, denoted by W' or $D(W)$, is the word whose jth symbol equals the length of the jth run of W, discarding the first and/or last run if it has length one. For example, $(12211)' = 22$ and $(121)' = 1$. Write ϵ for the empty word and set $\epsilon' = \epsilon$.

Say that a finite word of 1's and 2's is C^∞, or is a C^∞*-word*, if it is arbitrarily often differentiable. For example, 1212 is C^∞ and 12121 is differentiable but not C^∞.

If S is a finite subword of the Kolakoski sequence K, then S is differentiable and S' is either ϵ or a subword of K. Thus every finite subword of K is C^∞.

Definition. A C^∞ word W is *recurrent* (or *almost periodic*) if there is a positive integer n such that every C^∞ word of length at least n contains W as a subword.

Considerable effort has been spent trying to prove

Conjecture. Every C^∞-word is recurrent.

This would imply that the finite subwords of K are exactly the C^∞ words.

In this talk, we consider evidence for and against the conjecture.