Abstract for talk by Doug Weakley in Discrete Math Seminar
4:30 p.m. on Wednesday, September 29, in Kettler 119

Is every $C^\infty$-word recurrent?

The sequence $K = 12212112212112112212112112211...$ given by W. Kolakoski in 1965 can be described as an infinite sequence of 1’s and 2’s that begins with 1 and has the property that the length of the $j$th run of like symbols is equal to the $j$th symbol.

**Question.** What are the finite subwords of $K$?

**Definitions.** A finite word $W$ of 1’s and 2’s in which neither 111 nor 222 occurs is differentiable, and its derivative, denoted by $W'$ or $D(W)$, is the word whose $j$th symbol equals the length of the $j$th run of $W$, discarding the first and/or last run if it has length one. For example, $(12211)' = 22$ and $(121)' = 1$. Write $\epsilon$ for the empty word and set $\epsilon' = \epsilon$.

Say that a finite word of 1’s and 2’s is $C^\infty$, or is a $C^\infty$-word, if it is arbitrarily often differentiable. For example, 1212 is $C^\infty$ and 12121 is differentiable but not $C^\infty$.

If $S$ is a finite subword of the Kolakoski sequence $K$, then $S$ is differentiable and $S'$ is either $\epsilon$ or a subword of $K$. Thus every finite subword of $K$ is $C^\infty$.

**Definition.** A $C^\infty$ word $W$ is recurrent (or almost periodic) if there is a positive integer $n$ such that every $C^\infty$ word of length at least $n$ contains $W$ as a subword.

Considerable effort has been spent trying to prove

**Conjecture.** Every $C^\infty$-word is recurrent.

This would imply that the finite subwords of $K$ are exactly the $C^\infty$ words.

In this talk, we consider evidence for and against the conjecture.