THE DEPARTMENT OF MATHEMATICAL SCIENCES

Indiana University - Purdue University Fort Wayne

is pleased to present

Robert S. Womersley

School of Mathematics and Statistics
University of New South Wales, Sydney, Australia

Efficient Spherical Designs with Good Geometric Properties

Abstract

Spherical t-designs on the unit sphere $\mathbb{S}^d \subset \mathbb{R}^{d+1}$, introduced by Delsarte, Goethals, and Seidel (1977), are equal weight numerical integration rules that are exact for all polynomials of degree at most t on $\mathbb{S}^d$. This talk considers the calculation and properties of of spherical t-designs, in particular for $\mathbb{S}^2$ where most applications reside.

Bondarenko, Radchenko, and Viazovska (2013) proved that there exists a c_d such that spherical t-designs with N points exist for all $N \geq c_d t^d$, which is the optimal order. Moreover they showed that there exist such spherical designs that are well-separated (2014). The interest here is in efficient spherical designs with $N < t^d$.

The geometric properties of point sets on $\mathbb{S}^d$ can be characterised by their separation (twice the packing radius), their mesh norm (covering radius), and mesh ratio (covering radius / packing radius), amongst many other criteria. A common assumption arising in applications is that the the sequence of point sets is quasi-uniform, that is, their mesh ratios are uniformly bounded. The interest here is in sets of efficient spherical t-designs with small mesh ratios.

Examples of spherical t-designs on $\mathbb{S}^2$ with $N = t^2/2 + O(t)$ points and mesh ratio < 1.8 for $t = 1, \ldots, 311$ are available from:

`http://www.maths.unsw.edu.au/~rsw/Sphere/EffSphDes/`

These provide excellent sets of points for both numerical integration and approximation, for example by needlets.

Noon – 1:00, Wednesday, December 2, 2015. Location: KT 216

`http://ipfw.edu/departments/coas/depts/math/news/seminars.html`