THE DEPARTMENT OF MATHEMATICAL SCIENCES

Purdue University Fort Wayne

is pleased to present

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The Mastodon Theorem — 20 Years in the Making

Abstract

The *Mastodon Theorem* (PD., D. Legg, D. Townsend, 2002), establishes that the regular bi-pyramid (North and South poles, and an equilateral triangle on the Equator) is the unique, up to rotation, five-point configuration on the sphere that maximizes the product of all mutual distances. More generally, for $s \geq 0$, given a configuration of points $\{x_1, \ldots, x_N\}$ on the unit sphere in $\mathbb{S}^{n-1} \subseteq \mathbb{R}^n$, its *Riesz s-energy* is defined as

$$\sum_{1 \leq i < j \leq N} \frac{1}{\|x_i - x_j\|^s}, \quad s > 0; \qquad \sum_{1 \leq i < j \leq N} \log \frac{1}{\|x_i - x_j\|}, \quad s = 0.$$

The regular bi-pyramid then minimizes the *logarithmic energy* ($s = 0$ case) for five points on $\mathbb{S}^2$.

Optimal configurations that minimize the s-energy have broad applications in science, economics, information theory, etc. Rigorous proofs of optimality are extremely hard, though. Even the important Coulomb energy ($s = 1$) case for five points on the unit sphere in 3-D space was resolved only recently (2013) by Richard Schwartz in a 70-page manuscript utilizing a computer-aided proof.

In joint work with Oleg Musin initiated at ICERM, Brown University, we generalize the Mastodon Theorem to $n+2$ points on $\mathbb{S}^{n-1}$. The proof utilizes calculus and linear algebra techniques.

Noon – 1:00, (NEW DATE) Wed., Feb. 6, 2019. Location: Kettler 216

http://www.pfw.edu/departments/coas/depts/math/news/seminars.html