THE DEPARTMENT OF MATHEMATICAL SCIENCES

Indiana University - Purdue University Fort Wayne

is pleased to present

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Polynomial Techniques for Investigation of Spherical Designs

Abstract

Spherical designs were introduced in 1977 by Delsarte, Goethals and Seidel as a counterpart on the Euclidean sphere of the classical combinatorial designs.

A spherical τ-design is a finite set $C \subset \mathbb{S}^{n-1}$ such that the equality

$$\frac{1}{\mu(\mathbf{S}^{n-1})}\int_{\mathbf{S}^{n-1}} f(x)d\mu(x) = \frac{1}{|C|}\sum_{x\in C} f(x)$$

(where $\mu(x)$ is the usual Lebesgue measure) holds for all polynomials $f(x) = f(x_1, x_2, \ldots, x_n)$ of degree at most τ (i.e. the average of f over the set is equal to the average of f over $\mathbb{S}^{n-1}$).

An equivalent definition says that $C \subset \mathbb{S}^{n-1}$ is a spherical τ-design if and only if

$$\sum_{x\in C} f(\langle x, y\rangle) = |C|f_0 \tag{1}$$

holds for every $y \in \mathbb{S}^{n-1}$ and every real polynomial $f(t)$ of degree at most τ, where f_0 is the first coefficient in the Gegenbauer expansion of $f(t) = \sum_{i=0}^{k} f_i P_i^{(n)}(t)$.

In this talk we show how (1) can be used for some special (with respect to the design) points implying restrictions on the structure of the designs. This idea was first proposed and used by Fazekas-Levenshtein (1997) and our group since 1999.

In some cases (odd strength, odd cardinality, other conditions) this implies nonexistence results in the first open cases and in certain asymptotic process. Other applications lead to bounds (upper and lower) on the smallest and largest inner products. Still another consideration gives bounds on the covering radius of spherical designs.

Noon – 1:00, **Thursday, May 22**, 2014. Location: KT 218

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