Are Trees Graceful?

Abstract

Given a graph $G$ with $m$ edges, we assign labels to the vertices of $G$ from the set $\{0, \ldots, m\}$. We also assign to each edge of $G$ a label that is the absolute difference of the labels of its endpoints. We say that such a labeling is graceful, and the graph is graceful, if the vertex labels are distinct and the edge labels are distinct.

Ringel posed the following problem in 1963: Given a tree $T$ with $m$ edges, can the complete graph $K_{2m+1}$ be decomposed into $2m+1$ copies of $T$?

In an attempt to solve this, Rosa introduced in 1966 the concept of graceful labelings and certain other related types of labelings. The famous Kotzig-Ringel conjecture states that all trees are graceful. It has been shown by using computer programs that trees of order up to 35 are graceful. Bagga et al. investigated algorithms for generating all graceful labelings of some known classes of graceful graphs, including paths, cycles, and other unicyclic graphs. Data generated by such algorithms have led to the discovery of new properties of graceful labelings. We present a brief survey of these results.

Noon – 1:00, Wednesday, March 23, 2016. Location: KT 216